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THE COMPUTATION OF COLORIMETRIC PURITY*

By Irwin C. Priest
With the collaboration of L. B. Tuckerman, Herbert E. Ives, and F. K. Harris**

I. Experimental Definition of Purity[1]

Given any color of spectral hue (i.e., any color except a purple), it is possible to evoke the same color by a suitable mixture of heterogeneous radiant energy, which alone would evoke the hueless color commonly called "white,"[2] and homogeneous radiant energy of some wave-length depending on the given color.[3]

* Partial report No. 4 on cooperative investigations in colorimetry by the Bureau of Standards and the Munsell Research Laboratory. Published by permission of the Director of the Bureau of Standards, U. S. Department of Commerce, and the Director of the Munsell Research Laboratory.

**In addition to those mentioned as collaborators, the author takes pleasure in acknowledging the valuable assistance of Messrs. J. Keith Cromer and F. A. Durr who carried out most of the extensive routine computations, and Mr. C. L. Snow who prepared the figures (except Fig. 2).

[1] Sections I and II of this paper are intended to present only a condensed formal synopsis of fundamental relations rather than explanations or proofs. The intelligent reading of this paper presumes an acquaintance with the following previous papers:

Ives: "Transformation of Color-Mixture Equations," J.F.I. *180*, pp. 673-701; December, 1915, and J.F.I. *195*, pp. 23-44; January, 1923.

Troland: "Report of the Colorimetry Committee of the O. S. A. 1920-21," J.O.S.A. & R.S.I., *6*, pp. 527-596; August, 1922. (Separate copies of this committee report may be had at fifty cents per copy of F. K. Richtmyer, Bus. Mgr., J.O.S.A. & R.S.I., Cornell University, Ithaca, New York.)

[2] Whether we call this color "white" or "gray," is, *for our present purpose*, a matter of indifference. Usage on this point is not well established.

[3] Grassman: Pogg. Ann., *89*, p. 70; May, 1853, (or Grassmann's "Math. & Phy. Werke" *2*, part 2, p. 162). Popular expositions may be found in Rood's "Modern Chromatics," Abney's "Colour Measurement and Mixture," and Luckiesh's "Color and its Applications." See also Nutting, B. S. Bulletin *9*, pp. 1-5; 1913 and Priest: J.O.S.A. & R.S.I. *8*, pp. 173-200; January, 1923.

The natural method of specifying physically (objectively) the heterogeneous stimulus just described as competent to evoke the hueless color "white" is to specify the relative spectral distribution of energy. A further condition characteristic of the *normal* stimulus of white is that this distribution be (at least approximately) that of a complete radiator at some specified temperature.[4] The experimental determination of this temperature as a psycho-physical constant presents a problem of considerable difficulty which is not yet completely solved. While the selection of a particular temperature as standard is to a certain extent arbitrary,[5] still it would be unreasonable to select it outside of a certain range. The consensus of opinion appears to be that the temperature entitled with most reason to be chosen as standard lies between 5000° and 6000° absolute centigrade.[6] A decision as to the particular temperature to be chosen within these limits is not essential to our present purpose. The relations to be discussed in this paper will hold regardless of the particular temperature chosen as standard within these (or even wider) limits. We adopt the following definitions for the purposes of the present paper:

(1) ***The normal neutral stimulus is radiant energy of the spectral distribution of a complete radiator at some standard temperature between 5000° and 6000° absolute centigrade.*** For the sake of brevity this will generally be designated simply as the "neutral stimulus."

(2) ***The homogeneous stimulus is radiant energy of some particular wave-length, (or narrow range of wave-length, say 5 or 10 millimicrons).***

Consider a photometric field in one half of which is presented the given color, while the other half is illuminated by a mixture of the neutral and the homogeneous stimuli, this mixture being adjustable as regards: (1) the wave-length of the homogeneous stimulus; (2) the field brightness due to the homogeneous stimulus; (3) the field brightness due to the neutral stimulus. Let the mixture be adjusted until the given color is completely matched as regards all three attributes of color, namely: hue, brilliance, and saturation.[7]

Adopt symbols as follows:

Λ ≡dominant wave-length≡the wave-length of the homogeneous stimulus.

[4] Cf. "Report of Colorimetry Committee." J.O.S.A. & R.S.I., *6*, p. 564; August, 1922.

[5] Cf. Peddie: "Colour Vision" (London, 1922), pp. 16-17, 52-53, 108-109.

[6] Cf. Ives: Trans. I. E. S., *5*, pp. 196-198; April, 1910.
Priest: B. S. Sci. Pap. 417, pp. 256-258, particularly Fig. 15; August, 1921.

[7] See J.O.S.A. & R.S.I., *6*, p. 534; 1922.

B ≡brightness of the mixture field = brightness of sample field.

B_A ≡the component of field brightness due to the homogeneous stimulus.

B_w ≡the component of field brightness due to the neutral stimulus.

Then, *by definition*, we have

$$\text{Purity} \equiv p \equiv \frac{B_A}{B} \qquad [1]$$

$$\text{Impurity} \equiv i \equiv \frac{B_w}{B} \qquad [2]$$

(Over a wide range of brightness it is experimentally true that:[8]

$$B = B_A + B_w \qquad [3]$$

and

$$p + i = 1.00) \qquad [4]$$

II. THE MATCHING OF ANY COLOR BY THE COLOR EVOKED BY A MIXTURE OF THREE HOMOGENEOUS STIMULI[9]

Given any color (except some due to homogeneous or nearly homogeneous stimuli), it is possible to evoke the same color by a suitable mixture of three homogeneous (or nearly homogeneous) stimuli, the same three in different proportions serving to match all colors generally. Such a set of three stimuli is called a set of "primary stimuli." Experimentally there is considerable latitude in the choice of a set of primaries, so that many different sets of "working primaries" may be chosen with equal justification.

If the given color is itself evoked by homogeneous (or nearly homogeneous) energy then it may not be possible to completely match it by

[8] Ives: Phil. Mag., (6), *24*, pp. 845-853; Dec., 1912.
Priest: J.O.S.A. & R.S.I., *8*, pp. 198-200; Jan., 1924.

[9] Maxwell: "On the Theory of Compound Colours . . ." Phil. Trans., 1860. (Or Maxwell's "Scientific Papers," *I*, pp. 410-444.)
Koenig and Dieterici: Sitzb. der Akad. Berlin, pp. 805-829; July 29, 1886. (Or Koenig's "Gesam. Abhand. Physiol. Optik," pp. 60-87.)
Koenig and Dieterici: Zeit. für Phych. und Physiol. der Sinnesorgane *4*, pp. 241-347; 1892. (Or Koenig's "Gesam. Abhand," pp. 214-321).
Abney: "Researches in Colour Vision" (London, 1913) Chap. XV.
For general treatments see:
(1) Helmholtz' "Physiol. Optik," 2nd ed. (Leipzig, 1896), pp. 311-384.
(2) Köllner's "Die Stoerungen des Farbensinnes" (Berlin, 1912).
(3) Parsons' "Introduction to Study of Colour Vision" (Cambridge, 1915).
(4) Peddie's "Colour Vision." (London, 1922).
For a brief elementary treatment, see Watson's "Text-Book of Physics" (London, 1903) Chap. IX.

a color evoked by a mixture of radiant energy of three homogeneous components. The color evoked by the mixture may be less saturated than the given color. Thus, it is not possible, in general, to match spectrum colors by a mixture of three *real* working primaries. However, by a mathematical artifice, it is possible to *represent* the spectrum colors and *all* colors generally as matched by colors which would be evoked by the mixture of three *hypothetical* stimuli. These three *hypothetical* stimuli are called elementary stimuli. *They are introduced as a mathematical convenience in order to avoid negative quantities.*

Let ρ, γ and β be numbers proportional respectively to the amounts of three such elementary stimuli expressed in units such that

$$\rho' = \gamma' = \beta' \qquad [5]$$

where ρ', γ' and β' are a particular set of values of ρ, γ and β representing the mixture of stimuli required to evoke the same color as that evoked by the normal neutral stimulus (i.e., "white" or "gray").

Consider ρ_λ, γ_λ and β_λ as functions of λ in the sense that ρ_λ, γ_λ and β_λ represent values of ρ, γ and β required in order that the mixture of the three elementary stimuli may evoke the same sensation as homogeneous energy of wave-length λ, the energy being constant independent of λ.

Let E_λ represent, as a continuous function of wave-length (λ), the relative radiant energy (per unit wave-length) of the normal neutral stimulus.

From the condition imposed in equation 5 it follows that

$$\int_0^\infty \rho_\lambda E_\lambda d\lambda = \int_0^\infty \gamma_\lambda E_\lambda d\lambda = \int_0^\infty \beta_\lambda E_\lambda d\lambda \ . \qquad [6]$$

L_r, L_g and L_b are constants defined by the following conditions:

$$L_r + L_g + L_b = 1.000 \qquad [7]$$

$$L_r \rho_\lambda + L_g \gamma_\lambda + L_b \beta_\lambda = V_\lambda \qquad [8]$$

for *all* values of λ, where V_λ is the relative visibility of radiant energy of wave-length λ.

From 8, it follows that

$$L_r \int_0^\infty \rho_\lambda E_\lambda d\lambda + L_g \int_0^\infty \gamma_\lambda E_\lambda d\lambda + L_b \int_0^\infty \beta_\lambda E_\lambda d\lambda = \int_0^\infty V_\lambda E_\lambda d\lambda \qquad [9]$$

We now introduce r, g and b, *defined* as follows:

$$r \equiv \frac{\rho}{\rho + \gamma + \beta} \qquad [10]$$

$$g \equiv \frac{\gamma}{\rho + \gamma + \beta} \qquad [11]$$

$$b \equiv \frac{\beta}{\rho+\gamma+\beta} \qquad [12]$$

It is evident that

$$r+g+b=1.000 \qquad [13]$$

The obvious frame for the graphic representation of r, g and b pertaining to *any* color is the equilateral triangle of unit altitude (Fig. 1). Within such a triangle the sum of the three perpendiculars from any

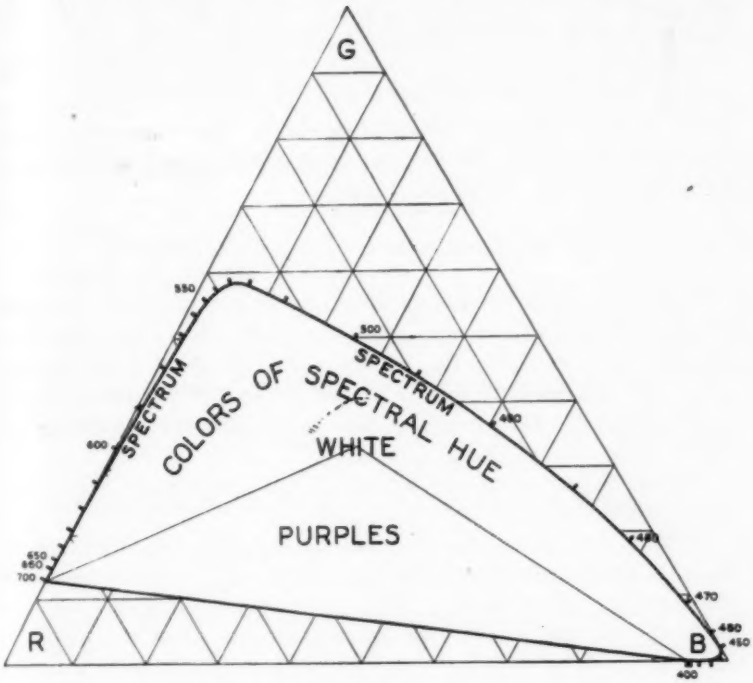

FIG. 1

point to the three sides equals unity. r, g and b are represented respectively by distances from the three sides of the triangle. If the points determined by the values of r, g and b for colors evoked by homogeneous radiant energy of various wave-lengths be plotted, their locus will be a curve represented approximately by the heavy line marked "Spectrum." The scale (700 to 400) along this curve gives the wave-lengths (in millimicrons) of homogeneous radiant energy competent to evoke the color for which the values of r, g and b are those indicated

by the location of the point in the triangle. By the condition of equation 5 the sensation gray will correspond to the center of the triangle, because the center is located by the condition

$$r = g = b \qquad [14]$$

If the values of r, g and b for any color are such that a straight line drawn *from* the center through the point located by r, g and b (and projected) intersects the curve marked "Spectrum," then λ for that color is given by the wave-length scale at said intersection.

The center of the triangle evidently corresponds to the condition p = zero. Any point on the curve marked "Spectrum" evidently corresponds to the condition p = unity (formula 1). The value of p corresponding to any other point can only be determined by consideration of L_r, L_g and L_b. (See equations 7 and 8.)

III Derivation of a Formula for Purity

Consider any color represented by a point (r, g, b) such that a straight line drawn from the center through the point and projected will intersect the curve marked "Spectrum." Let R, G and B represent the coordinates of this intersection. Let C represent the *least* of the three coordinates R, G and B. Let c represent the least of the three coordinates r, g and b.

A formula for p as defined (formula 1) may now be derived by the following argument:

(1) It is first to be observed that the *definition* of p (formula 1) amounts to *assigning* the value unity to p for all colors evoked by homogeneous radiant energy. (That is, *by this definition*, $p = 1.000$ for all of the spectral colors.) It appears, however, from the spectrum curve in Fig. 1, that a mixture of finite amounts of *all three* elementary stimuli is required to evoke the same color as is evoked by homogeneous radiant energy in the case of certain wave-lengths. Now since R, G and B are all positive and finite, and C is the least of R, G and B, it is apparent that the sensation evoked must, on an *absolute* basis, contain an element of gray, because each of the two stimuli represented by coordinates greater than C would contain an amount equal to C, and, by definition (5), the three stimuli in equal parts evoke the gray sensation. [The locus of (imaginary) stimuli containing no fraction competent to evoke gray is *the sides of the triangle* (Fig. 1).]

We thus arrive at a new concept, that of *absolute impurity*[10] which we represent by the symbol i'; and we define i' for any color as follows:

$$i' = \left\{ \frac{\text{The component of brightness due to the joint action of the three stimuli each taken in the amount, } c}{\text{The brightness due to the joint action of the three stimuli taken respectively in the amounts } r, g, \text{ and } b.} \right\} \quad [15]$$

The absolute impurity of the spectrum color of coordinates R, G, B is given by the special case of formula 15 substituting R, G, B and C respectively, for r, g, b and c.

(2) It is obvious that the numerator in formula 15 is equal to

$$Kc(L_r+L_g+L_b)$$

and the denominator is equal to

$$K\,(rL_r+gL_g+bL_b)$$

where K is a constant.

Consequently formula 15 reduces to

$$i' = \frac{K\,c\,(L_r+L_g+L_b)}{K\,(rL_r+gL_g+bL_b)} \quad [16]$$

Substituting formula 7 in 16 and cancelling K, we obtain

$$i' = \frac{c}{(rL_r+gL_g+bL_b)} \quad [17]$$

Accordingly, the absolute purity for any color (p_c') is given by

$$p_c' = 1 - \frac{c}{(rL_r+gL_g+bL_b)} \quad [18]$$

In similar manner, the *absolute purity* for the spectrum color (p_C') is given by

$$p_C' = 1 - \frac{C}{(RL_r+GL_g+BL_b)} \quad [19]$$

(3) Since, in the definition of p (as experimentally measured in "monochromatic analysis," formula 1), we have arbitrarily assigned the value unity to the purity of the spectrum color, it is apparent that

$$p = \frac{p_c'}{p_C'} \quad [20]$$

[10] This "absolute impurity" of course presupposes a given set of "elementary stimuli." It is not "absolute" if a change of "elementary stimuli" is involved.

or

$$p = \frac{1 - \dfrac{c}{rL_r + gL_g + bL_b}}{1 - \dfrac{C}{RL_r + GL_g + BL_b}} \qquad [21]$$

IV Comparison of Various Formulas and Graphs

I derived the above formula (21) in May 1923 by substantially the argument just given. So far as I am aware the only previously published algebraic formulas leading readily to an explicit solution for p in terms of r, g, b, R, G, B, L_r, L_g, L_b are those given by H. E. Ives.[11] A graphic solution has also been given by Ives.[12]

A conversion graph and table due to E. A. Weaver, have been published in the Report of the Colorimetry Committee, Optical Society of America.[13] We shall now compare the above formula (21) with:

(1) The previous formula of Ives;[14]
(2) The graphic solution of Ives;[15]
(3) The data given by Weaver.[16]

1. Comparison of the above formula with Ives' formula

Soon after the above formula (21) was derived it was found that it did not satisfactorily check Ives' graphic method except for values of λ between 500 and 600 millimicrons.[17] The formula was therefore submitted to Dr. Ives for criticism. He expressed the opinion that the formula was apparently valid although novel in form,[18] but suggested that it be compared with his own previous algebraic formula. Dr. Ives kindly supplied an explicit formula for p derived from his published formulas.[19]

[11] J.F.I., *180*, pp. 673-701; Dec., 1915.

[12] J.F.I., *195*, pp. 23-44; Jan., 1923.

[13] J.O.S.A. & R.S.I., *6*, pp. 527-596; August, 1922. See particularly Fig. 9, p. 575, and Table 14A, pp. 586-587.

[14] J.F.I, *180*, p. 684; Dec., 1915.

[15] J.F.I., *195*, Fig. 9, p. 35; Jan., 1923.

[16] J.O.S.A. & R.S.I., *6*, pp. 586-587; August, 1922.

[17] See Sec. IV-2 below for details.

[18] "This formula" [21, in the present paper] "appears to be valid, although in a form which differs from any heretofore proposed, in the use of 'c' and 'C,' the 'least coordinates' "—From letter of Herbert E. Ives to the Director of the Bureau of Standards, October 1, 1923.

[19] J.F.I., *180*, p. 684; Dec., 1915.

Stated in terms of the symbols of the present paper this formula[20] is

$$\left.\begin{aligned} p &= \left(\frac{r-g}{rL_r+gL_g+bL_b}\right)\left(\frac{RL_r+GL_g+BL_b}{R-G}\right) \\ \text{or} \\ p &= \left(\frac{b-r}{rL_r+gL_g+bL_b}\right)\left(\frac{RL_r+GL_g+BL_b}{B-R}\right) \\ \text{or} \\ p &= \left(\frac{b-g}{rL_r+gL_g+bL_b}\right)\left(\frac{RL_r+GL_g+BL_b}{B-G}\right) \end{aligned}\right\} \quad [22]$$

An extensive comparison made by substituting identical numerical data in the two formulas, 21 (Priest) and 22 (Ives) showed that they always gave equal values for p; and created a strong presumption that they were mathematically identical, although very different in form. That they are mathematically *identical* has since been demonstrated by simple algebraic transformations by Dr. L. B. Tuckerman[21] and, later but entirely independently, by Mr. F. K. Harris. Dr. Tuckerman has shown not only that formula 21 can be derived from formula 22 but also that all of the following formulas analogous to 21 can be derived from 22, namely:

$$p = \frac{1-\dfrac{r}{rL_r+gL_g+bL_b}}{1-\dfrac{R}{RL_r+GL_g+BL_b}} \quad [23a]$$

$$p = \frac{1-\dfrac{g}{rL_r+gL_g+bL_b}}{1-\dfrac{G}{RL_r+GL_g+BL_b}} \quad [23b]$$

$$p = \frac{1-\dfrac{b}{rL_r+gL_g+bL_b}}{1-\dfrac{B}{RL_r+GL_g+BL_b}} \quad [23c]$$

Further, he has shown that these formulas may all be comprehended in a simpler *general* expression. Dr. Tuckerman's simplified general formula is

$$p = \frac{3-\dfrac{1}{rL_r+gL_g+bL_b}}{3-\dfrac{1}{RL_r+GL_g+BL_b}} \quad [24]$$

(See Appendix to this paper).

[20] Letter, Ives to Director, Bureau of Standards, Oct. 1, 1923.

[21] See appendix, this paper.

At first sight it might appear that this formula would be more simple than any of the others to apply in numerical computation. However, it is actually found that, in many cases, the value of either the numerator or the denominator may be so nearly zero as to make its accurate evaluation impossible. (Cf. second paragraph just below.) This circumstance precludes the perfectly general numerical application of this formula for the practical purpose of computing p.

Ives' formula 22 or Tuckerman's formula 24, is to be regarded as the general case. Formula 21 is a special case. The original independent derivation of 21 and 22 by quite different lines of argument may be regarded as confirmatory evidence of the validity of both derivations.[22]

In choosing between the three forms of formula 22 for the application of given numerical data one will naturally choose that one in which the denominator of the second factor (and the numerator of the first factor) is the greatest (without regard to sign), for this will obviously yield the highest relative accuracy in the computed value of p.[23]

2. COMPARISON OF THE FORMULAS WITH IVES' GRAPHIC SOLUTION

The final result of Ives' graphic solution for p is exemplified in Fig. 9, p. 35, *Jour. Frank. Inst.*, Jan. 1923, which is reproduced as Fig. 2 in the present paper. Dr. Ives has kindly supplied me a large scale (altitude of triangle = 36.5 cm) copy of this figure.

Fig. 3 shows a graphic synopsis of the actual comparison of values of p obtained respectively from:—

(1) Ives' graph (large scale copy just mentioned);

(2) Ives' formula (formula 22 of the present paper in its most accurate form);

(3) The formula proposed in the present paper (21).

These results were obtained in the following way:

(1) Points were taken for a number of values of Λ on each of the loci of purity, 0.8, 0.6, 0.4, 0.2, 0.1, and 0.05 in Ives' graph.

(2) The coordinates r, g, b, for each of these points were read from the graph.

[22] "It is gratifying that formulas arrived at through such entirely independent lines of thought and which look so different are identical in the value they give"—Letter, Ives to Priest, Feb. 27th, 1924, referring to the numerical agreement of values of p computed respectively by formulas 21 and 22.

[23] Casual inspection of these formulas (22) might suggest the mathematical possibility of negative values of p, for it is apparent that the numerator of the first factor and the denominator of the second factor may be negative. Negative values of p would have no physical meaning. However, these factors are always of the *same* sign whether positive or negative. Consequently the computed value of p is always positive.

(3) The coordinates R, G, B, were read from the graph at the intersections of the locus of purity = 1.0 with straight lines drawn from the center ($p = 0.0$) through the above mentioned points (r, g, b).

(4) The values of L_r, L_g, L_b were taken as given by Ives.[24] These values are:

$$L_r = 0.568$$
$$L_g = .426$$
$$L_b = .006$$
$$L_r + L_g + L_b = 1.000$$

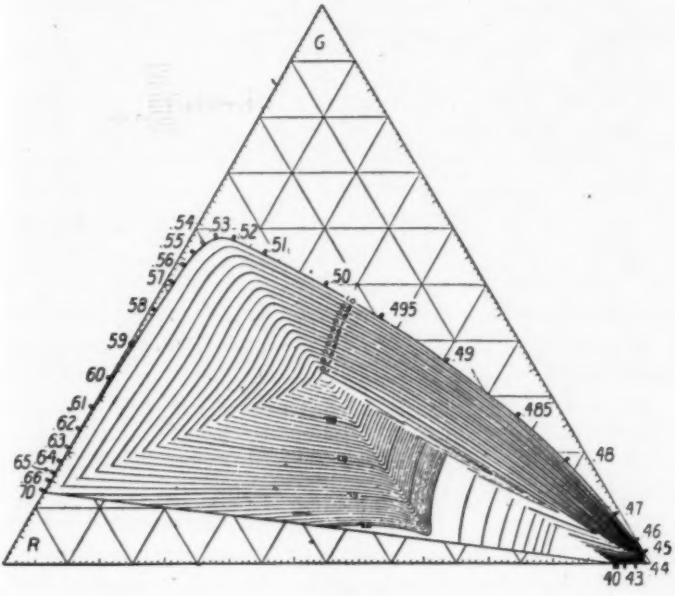

FIG. 2.
(Reproduction of Fig. 9 from Ives' Paper, p. 35, J.F.I., Jan. 1923)

(5) The just mentioned numerical data for r, g, b, R, G, B, L_r, L_g, L_b were entered in Ives' formula (22) and in the formula proposed in the present paper (21); and the two values of p were computed by these two formulas.

Fig. 3 is to be understood as follows:

(1) The abscissa represents dominant wave-length (Λ).

(2) The ordinate represents purity (p).

[24] J.F.I., *195*, p. 40; Jan., 1923.

(3) The small solid circles result from Ives' formula (22).

(4) The large open circles result from the formula proposed in this paper (21).

(5) Points connected by lines all result from data taken from a locus of constant purity in Ives' graph; and the value of *p from the graph* is indicated by a number attached to each line.

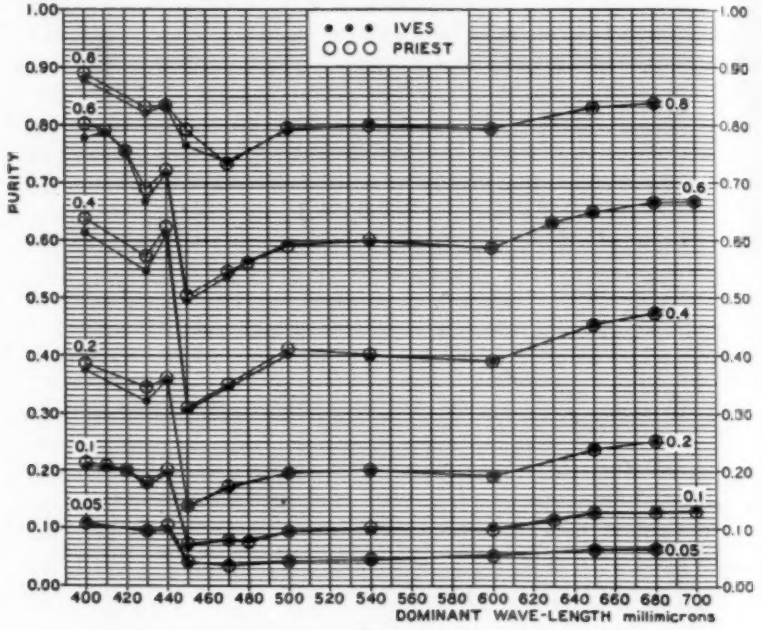

FIG. 3*

The following conclusions are obvious from an inspection of Fig. 3.

(1) The formulas being really identical,[25] the small discrepancies in the numerical results are to be attributed to rejection errors in computation. The data can not be scaled from Ives' graph with sufficient accuracy to give *exact* agreement in the results of the two formulas.

(2) Values of purity read from the purity loci on Ives' graph are in fair agreement with values given by the formulas (21 and 22) *only for the limited range of dominant wave-length between about 500 and 600 millimicrons.*

(3) For dominant wave-lengths greater than 600 millimicrons, values of purity read from the graph are less than the values found by

[25] Cf. Sec. IV-1, this paper.

the formulas; and the discrepancy increases regularly with increasing wave-length.

(4) Over a narrow wave-length region at about 470 millimicrons, values read from the graph are greater than those given by the formulas.

(5) For wave-lengths less than 450 millimicrons, values read from the graph are less than those given by the formulas.

(6) The discrepancies between graph and formula are particularly large for dominant wave-lengths less than 440 millimicrons.

(7) The above relations hold, to some extent, for all values of purity; but the relative errors in purity increase notably with decreasing purity.

When the above conclusions were submitted to Dr. Ives, he commented as follows:[26]

"The method used in the formulae sacrifices the luminosity curve to the sensation curves, the graph sacrifices the sensation curves to the luminosity curve. If the sensation and luminosity curves are both correct and therefore consistent, the two methods must give identical results, and the problem will not be completely solved until we have data which may be handled with like results by either method."

In effect, it appears that Ives' graphic method amounts to defining L_r, L_g and L_b by equation 9 in the present paper rather than by equation 8. Now although equation 9 necessarily follows from 8, the converse is not true. Examination of Ives' Fig. 12, p. 39, *Jour. Frank. Inst.*, Jan. 1923 will elucidate this point.

It appears then that:

(1) The theoretical methods and formulas for the computation of purity from trilinear coordinates (r, g, b) and the "luminosity coefficients" (L_r, L_g, L_b) are established.

(2) Further experimental data on ρ_λ, γ_λ, β_λ, V_λ and L_r, L_g, L_b *as defined in equation 8* are required in order to apply these methods and formulas to the purpose of obtaining *very accurate* values of p.

3. COMPARISON OF THE DATA GIVEN BY WEAVER WITH THE RESULTS OF THE ABOVE FORMULAS

Weaver's data for the relation between p and the trilinear coordinates (r, g, and b) are given in detail in Table 14A, pp. 586-587, J.O.S.A. & R.S.I., *6*; August, 1922. This table appears to have been constructed by computing r, g, b, (using the symbols of the present paper) for a great number of values of Λ and p.[27] That is, the problem of conversion

[26] Letter, Ives to Priest, Feb. 27, 1924.

[27] See J.O.S.A. & R.S.I., *6*, pp. 582-585; August, 1922.

has been approached from the direction opposite to that dealt with above; and no explicit algebraic solution for p is given. However, Weaver's tabular data being put in graphic form, (as for example in Fig. 9, p. 575, J.O.S.A. & R.S.I., August, 1922), one may obtain (by interpolation), p for any given values of r, g, b. The values of L_r, L_g, L_b pertaining to this table were apparently as follows:

$$\left.\begin{aligned} L_r &= 0.370 \\ L_g &= 0.617 \\ L_b &= 0.012 \\ L_r+L_g+L_b &= 0.999 \end{aligned}\right\} \quad \begin{array}{l}\text{(See J. O. S. A. \& R. S. I.,} \\ \text{6, p. 551 and p. 582;} \\ \text{August, 1922.)}\end{array}$$

Of course these values of r, g, b and L_r, L_g, L_b refer to different elementaries than those in Ives' graph. This however, is of no moment in the present discussion. We should still expect equation 22 (or 21) to be satisfied by Weaver's data for r, g, b and L_r, L_g, L_b.

We now proceed to inquire as to how accurately the equation is satisfied by Weaver's data in general. The results of this inquiry are exhibited in Fig. 4, which has been constructed in a manner analogous to Fig. 3.[28] The values of r, g, b for given values of Λ and p were taken from Weaver's Table 14A.[29]

The formula was applied in the three forms given in equations 23a, 23b, and 23c, which have been shown to be identical with 22.[30] The small circles result from computation by 23a; the medium sized circles result from 23b; and the largest circles from 23c. The solid and heavy line circles represent the special case of formula 23 which is identical with 21. The small discrepancies are of course to be attributed to the different effects of rejection errors of numerical data in the three forms. (The numerical data were taken in each case for r and b exactly as given by Weaver in his Table 14A; and g was taken as $1.00-r-b$. The values of L_r, L_g, L_b were used as cited just above.) The notable discrepancies at $\Lambda = 580$ are readily accounted for on this basis. At this wave-length $\dfrac{G}{RL_r+GL_g+BL_b}$ is 0.925; and $\dfrac{g}{rL_r+gL_g+bL_b}$ for $p=0.10$ is 0.996. Since G and g are given only to 3 significant figures, it is apparent that the computed value of p will be subject to a relatively large uncertainty.

[28] See IV-2 above.

[29] J.O.S.A. & R.S.I., 6, pp. 586-587; August, 1922.

[30] See Sec. IV-1 above.

Points connected by lines result from values of r, g, b for a constant value of p in Weaver's data; and this constant value is indicated by a number attached to the line in each case.[31]

The following conclusions are drawn from examination of Fig. 4:

(1) Ives' formula[32] (23 or 22 this paper) is accurately satisfied by Weaver's data *except for values of dominant wave-length less than about 470 millimicrons.*

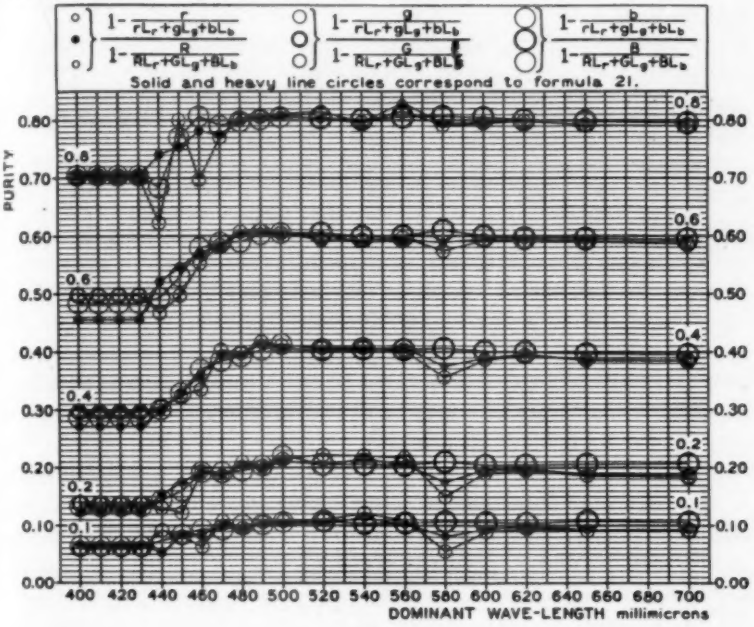

FIG. 4

(2) For values of dominant wave-length less than 470 millimicrons, p by the formula is less than p from Weaver's table, the discrepancy increasing continuously with decreasing wave-length to 430 and then remaining constant between 430 and 410 millimicrons.

(3) These relations hold in similar form for all values of p.

Examination of Weaver's Fig. 9[33] suggests a reason for this discrepancy. Wave-length 470 millimicrons appears to represent a "critical

[31] Note that Weaver's tabulation is actually given in terms of $(1-p)$ in *per cent*, not directly in terms of p.

[32] Or the formula derived above (equation 21) which is identical with it.

[33] J.O.S.A. & R.S.I., *6*, p. 575; August, 1922.

point." For wave-lengths less than 470, R is *given* as zero; and G is very small. The relative accuracy with which the true values of R and G are given is therefore very low. It is suggested then that the discrepancy may be due merely to the effect of rejection errors in computation.[34]

V CONCLUSION

The following conclusions are drawn from the comparative data presented above:

(1) The formulas for colorimetric purity in terms of the "trilinear coordinates" (r, g, b) and the "luminosity coefficients" (L_r, L_g, L_b) derived independently by quite different arguments respectively by Ives (formula 22) and Priest (formula 21) are identical.

(2) Ives' graphic solution as actually given in 1923 (Fig. 9, J.F.I., p. 35) is not in satisfactory accord with the formula previously proposed by him and verified (in a special case) in the present paper (formula 22); but this discrepancy is apparently due to lack of consistent data on the functions ρ_λ, γ_λ, β_λ on one hand and V_λ on the other. In other words, if such consistent data were applied in both cases, it appears that Ives' algebraic formula and his graphic solution would lead to the same result. The obtaining of such consistent data presents an *experimental* problem which must be solved before much further progress can be made in this subject.

(3) The conversion table for p on one hand and r, g, b together with L_r, L_g, L_b on the other[35] given in the Report of the Colorimetry Committee, O.S.A., 1920-21,[36] and attributed to E. A. Weaver[37] is in accurate accord with the above mentioned formulas except for values of dominant wave-length less than 470 millimicrons; and it appears probable that the discrepancies for these shorter wave-lengths may be accounted for by lack of consistency in the original data, used by Weaver, or perhaps by a mere lack of arithmetical or graphic accuracy in the construction of this conversion table.[38]

It then appears that, from a theoretical point of view (the principle of the additivity of brightnesses being assumed), the methods and

[34] In a letter of July 30, 1924, addressed to the author after reading this paper in manuscript Mr. Weaver expresses the opinion that these deviations are to be attributed to inconsistent data for the "excitation-curves" and the "visibility-curves." (Cf. IV-2 above.) I.G.P. Aug. 4, 1924.

[35] See first paragraph, p. 585, J.O.S.A. & R.S.I., August, 1922.

[36] Table 14A, pp. 586-587, J.O.S.A. & R.S.I., August, 1922.

[37] J.O.S.A. & R.S.I., *6*, p. 527 and p. 548; August, 1922.

[38] Cf. last paragraph of Sec. IV above.

formulas for the computation of colorimetric purity are satisfactorily established in form for practical application. This practical application, in the fullest sense, however, requires more reliable and consistent experimental data on

$$\rho_\lambda,\ \gamma_\lambda,\ \beta_\lambda,\ L_r,\ L_g,\ L_b \text{ and } V_\lambda$$

The question as to whether or not the basic premises assumed by Ives and Priest in deriving the formulas were valid can only be decided by a comparison of *experimental* values of p with values *computed by the formulas*. This comparison will involve in a complex way the experimental data mentioned in the preceding paragraph. The present paper has dealt only with the mathematical question of the *conversion* of data. A paper by Priest, Gibson, and Munsell, now in course of preparation,[39] will deal in a preliminary way with the direct comparison of experimental values of purity (as defined in formula 1 of the present paper) and values computed by the formulas.

NATIONAL BUREAU OF STANDARDS,
MARCH-AUGUST, 1924.

APPENDIX

PROOF THAT THE PURITY FORMULAS OF IVES AND PRIEST ARE IDENTICAL, AND DERIVATION FROM THEM OF A SIMPLER EXPRESSION FOR PURITY.

BY L. B. TUCKERMAN

Ives' equations (Formula 22) may be written

$$p=q\frac{RL_r+GL_g+BL_b}{rL_r+gL_g+bL_b}$$

$$\text{where } q=\frac{r-g}{R-G}=\frac{g-b}{G-B}=\frac{b-r}{B-R}$$

These three equations are not independent since any one follows directly by composition from the other two. They represent therefore only one independent condition imposed on $b, g, r, B, G,$ and R.

Priest's equation (Formula 21) may be written

$$p=\frac{rL_r+gL_g+bL_b-c}{RL_r+GL_g+BL_b-C}\times\frac{RL_r+GL_g+BL_b}{rL_r+gL_g+bL_b}$$

The two formulas are identical if

$$q=\frac{rL_r+gL_g+bL_b-c}{RL_r+GL_g+BL_b-C}$$

Defining x by

$$q=\frac{x-r}{x-R}$$

[39] Advance abstract already published. See proceedings Eighth Annual Meeting, Optical Society of America, J.O.S.A. & R.S.I., *8*, pp. 28-29; January, 1924. A further communication will be made at Ninth Annual Meeting, Boston, October, 1924. The complete paper will appear later as a B. S. Sci. Paper.

it follows by composition that:

$$q=\frac{x-r}{x-R}=\frac{x-g}{x-G}=\frac{x-b}{x-B}$$

and also:

$$q=\frac{L_r(x-r)+L_g(x-g)+L_b(x-b)-(x-c)}{L_r(x-R)+L_g(x-G)+L_b(x-B)-(x-C)}$$

where L_r, L_g, and L_b are arbitrary coefficients and c and C any pair of numbers satisfying

$$q=\frac{x-c}{x-C}$$

In particular c and C may represent r and R, g and G or b and B, or may be both equal to x, which gives a symmetrical expression for p.

Since $L_r+L_g+L_b=1$
this reduces to

$$q=\frac{rL_r+gL_g+bL_b-c}{RL_r+GL_g+BL_b-C}, \qquad \text{Q. E. D.}$$

The foregoing proof is independent of any limitations on r, g, b, R, G, and B, not implicitly contained in Ives' formula (22).

We know however that

$$r+g+b=1$$
$$\text{and} \qquad R+G+B=1$$

Then by composition

$$q=\frac{x-r+x-g+x-b}{x-R+x-G+x-B}$$
$$=\frac{3x-1}{3x-1}$$

This can only be true in general provided

$$x=1/3$$

The condition

$$q=\frac{1/3-b}{1/3-B}=\frac{1/3-g}{1/3-G}=\frac{1/3-r}{1/3-R}$$

is the analytical expression of the fact that the points (r, g, b) and (R, G, B) are collinear with the center $(1/3, 1/3, 1/3)$ of the color triangle. Substituting this value, $x=1/3$, for c and C in the formula, gives the symmetrical form

$$p=\frac{3-\dfrac{1}{rL_r+gL_g+bL_b}}{3-\dfrac{1}{RL_r+GL_g+BL_b}}.$$

Triple Sphere Spherometer.—The spherometer consists of three ball bearings, selected to have equal diameters, placed on a plane and touching each other. A fourth sphere, to be measured, is very lightly covered with vaseline, rested on the other three, and removed. From the distances between the points of contact and the diameter of the balls the diameter of the fourth sphere can be computed to about 0.2 per cent. Other arrangements of the same principle differing in mechanical details are described. [M. A. Biot, Annales de la Société scientifique de Bruxelles, *43*-I, p. 96; 1923] P. MERRITT

THE AVERAGE QUANTITY OF ULTRAVIOLET LIGHT ENERGY REQUIRED TO RENDER DEVELOPABLE A GRAIN OF PHOTOGRAPHIC SILVER BROMIDE

By P. S. Helmick

Various investigators have measured the amount of light-energy in the visible spectrum required to render developable a light-sensitive grain of silver-bromide as found in the emulsion of an ordinary film or plate.

Eggert and Noddack[1] in 1921 found that 280 quanta of wave-length 4078A were required to produce a developable grain of silver bromide. Later measurements by Noddack, Streuber and Scheffers[2] with 4360A showed that from 3600 to 52000 quanta are required, depending upon the amount of exposure given the plate.

In 1922 the author[3] found that about 2×10^7 incident quanta of mean wave-length 5490A are required to render developable a grain of silver bromide.

A series of measurements in the same year by Jones and Schoen[4] on plates of unit density and $\gamma = 1$ with wave-length 4200A indicated that with grain diameters varying from ¼ micron to 10 microns, the number of incident quanta per grain ranged from 2 to 31000, depending upon the size of the grain.

It would seem interesting to extend these measurements into the near ultraviolet, especially as a resonant frequency of silver bromide presumably exists in the region of 1900A.[5] The present paper gives measurements on the average quantity of light-energy required to render developable a grain of photographic silver bromide for the wave-lengths 3650A, 3131A, 2653A, and 2537A.

The experimental procedure was carried out in practically the same manner as in the earlier work[6] with wave-length 5490A: a piece of commercial photographic film which furnished the silver bromide grains was first exposed to a known quantity of monochromatic light-energy; and was then developed, fixed and washed; and the number of grains

[1] Sitz. Akad. Wiss. Berlin. P. 631; 1921.
Physikal. Zeit. *22*, p. 673; 1921.
[2] Sitz. Akad. Wiss. Berlin, P. 210; 1922.
[3] J.O.S.A. & R.S.I. *6*, p. 998; 1922.
[4] J.O.S.A. & R.S.I. *7*, p. 213; 1923.
[5] J.O.S.A. & R.S.I. *6*, p. 998; 1922.
[6] Loc. cit.

reduced was finally determined by direct microscopic counting. It was thus possible to obtain the value of the energy required to cause a grain of silver bromide to become developable.

LIGHT-SOURCE AND ENERGY MEASUREMENTS

The source of ultraviolet light used for the work consisted of a quartz Heraeus mercury-lamp operated with a constant current of 3 amperes and a Hilger ultraviolet monochromatic illuminator.

The intensity of the ultraviolet radiation from the illuminator was measured by comparison with the radiation from a Bureau of Standards certified lamp, using a Hilger linear thermolipe with receiving surfaces 1.5 mm in width.

In order to provide that the image of the slit of the monochromator should cover the whole receiving surface of the thermopile, a slit width of 1.5 mm was used on the monochromator. The matter of the spectral impurity introduced by this large slit-width will be considered in a following paragraph.

Furthermore, to ensure that the desired line in the spectrum should be properly directed upon the thermopile strips, it was found convenient to mount a small square of uviol glass in a fibre holder so that the glass could be inserted in the arm of the monochromator to lie in the plane normally occupied by the thermopile strips and be observed with a micrometer microscope attached to the arm of the monochrometer. The microscope was adjusted after placing the thermopile in position in the monochromator by focussing the microscope upon the receiving strips of the thermopile and recording the relative location of the strip image on the micrometer eyepiece. With the monochromator wave-length drum set at the desired wave-length and with the uviol glass in place, the adjustment of the instrument was always checked by making sure that the desired spectral line of the mercury arc occupied the same position on the micrometer scale as the thermopile image.

The galvanometer employed in conjunction with the thermopile was a four coil astatic needle instrument which had been used by Trowbridge.[7] Its coils were connected so that their combined resistance amounted to 15.5 ohms, a value which compared favorably with the 13 ohm resistance of the thermopile. The magnet system was constructed with 8 pieces of watch hairspring averaging 1.2 mm long mounted upon a glass staff in two groups of four each. The total weight of the moving system was 8 mgm. The shielding provided for the in-

[7] Phys. Rev. *27*, p. 282; 1908.

strument (besides the soft iron blocks in which the coils were imbedded) consisted of three concentric iron shields whose combined thickness totalled 3.5 cm, and whose weight equalled 35 kgm. Two control magnets were used with the instrument, the one used for the preliminary adjustment was located inside the inner shield, while the other which provided a final adjustment was placed between the middle and the inner shield. It was found that the adjusting of the galvanometer was greatly facilitated by the attachment of a slow motion screw to the fine-adjustment control magnet. A potentiometer could be connected to the circuit at any time in order to obtain the figure of merit of the galvanometer.

The characteristics of the galvanometer as first set up were as follows:

Period ..... 44 sec.
Sensitivity at 2 m scale distance ..... 4×10^{-10} amp/mm.
Deflection for 1 candle-meter on thermopile ..... 1500 mm.

In actual practice it was found that an average sensitiveness of 9×10^{-10} amp/mm could be used, giving the advantage of a shorter period.

The drift of the galvanometer was not large, for all electrical connections of the thermopile and galvanometer were soldered and thermally insulated. By taking ten radiation readings for each intensity measurement, the relative error of the mean amounted to about 0.3 per cent.

The intensity of radiation incident upon the film could be varied by the introduction of concentric circular diaphragms between the Cornu prism and the collimator lens of the monochromator. The largest diaphragm (38.2 mm) was of slightly smaller diameter than the aperture of the instrument, while the smallest diaphragm made use of was No. 78 of a twist drill gauge (0.406 mm). It was assumed that there were no serious zonal aberrations in the lens, and that as a consequence the intensity of the transmitted light was proportional to the area of the diaphragm.

It was thus possible to measure the light intensity with the thermopile when the largest diaphragm was in place, and then to reduce the intensity to 1/880 of its measured value with diaphragm No. 78 for exposure of the photographic film.

Exposures, which ranged from 2 to 14.4 seconds, were made by hand with a sliding shutter, while listening to a watch ticking fifths of seconds.

SPECTRAL ERRORS

It has been previously mentioned that a great illuminator slit-width of 1.5 mm was used throughout the entire work in order that the full width of the thermopile strips should be covered by the slit-image of the monochromator. Such a large slit-width might be considered to introduce excessive spectral errors, so these errors will now be investigated.

Consider first the errors due to pure spectral overlapping of the array of more or less coincident images of this finite slit. The following table indicates the range of calculated wave-lengths which will be found in an infinitely narrow vertical strip at the middle of the spectral image.[8] In counting the number of developed centers in the photographic film after its exposure to light, only those grains were counted which lay in a narrow vertical strip at the middle of the slit image. Thus the table gives the extent of the spectral impurities present in the light incident upon the measured portion of a film when the drum of the monochromator registers a certain wave-length. In Fig. 1 these bands of impurity are graphically illustrated in connection with a photograph of the mercury arc used in the work.

Wave-Length Setting	Extent of Impurities
2537A	2505A—2569A
2653A	2605A—2701A
3131A	3053A—3209A
3650A	3518A—3782A

Due to the emission in the mercury arc, the 3131A line is the only one in this region, so that the spectral image is quite pure for this line. Next in purity to the 3131A setting is probably the 2537A setting, for photographs of the particular arc used show only the faintest trace of a line at 2524A(?)[9] and also a faint line at 2564A(?).

The 3650A setting gives that prominent group,[10] together with nine or ten very faint lines surrounding it of negligible intensity as compared to the central line.

The 2653A setting includes very faint lines at 2625A and 2675A a line of low intensity at 2640A, and a line of slightly greater intensity at 2600A. Dingle[11] gives the respective intensities of the 2653A group as (2652A) 5, (2653A) 4, and (2655) 4, and the 2640A and 2699A lines

[8] This particular assumption is made because the area on the photographic film chosen for microscopic measurement always corresponded to this condition.

[9] Wave-lengths are taken from Dingle, Proc. Roy. Soc. *100A*, p. 167; 1921.

[10] Loc. cit.

[11] Loc. cit.

as 3, but judging by the relative blackenings on the plate produced by the two lines 2640A and 2653A, the intensities of the 2640A and the 2699A lines are small compared to the 2653A group.

Thus but small spectral inaccuracies of overlapping are introduced by this large slit-width.

The other type of error entering into spectral measurements, which is less susceptible to calculation than the foregoing overlapping error is caused by the reflection of stray wave-lengths in the interior of the illuminator.

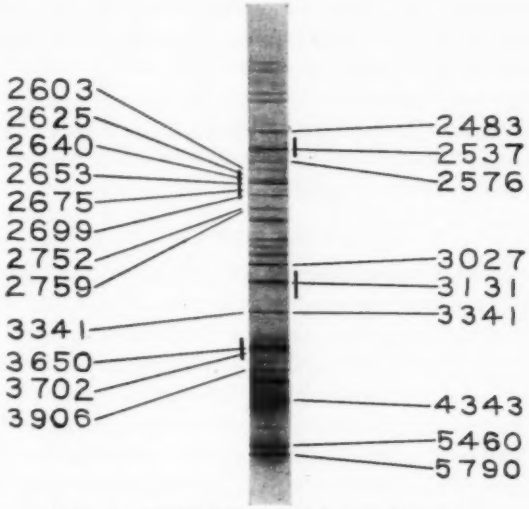

FIG. 1. *The slit-width error of the monochromator.*

In order to obtain data upon the matter of this stray light, a large Hilger spectrograph was placed to receive the light monochromatized by the ultraviolet illuminator. Various diaphragms were placed in the parallel light between the collimator and the prism of the monochromator, and with the wave-length drum set at a certain wave-length a series of equally timed exposures was given a photographic plate with different intensities of incident light on the plate. After radiometric measurements had been made on the different lines of the mercury arc for which wave-length settings on the monochromator had been made, the following values were obtained showing the relative energies of extraneous wave-lengths present at the emergent slit of the illuminator when the wave-length drum of the illuminator registered a given

setting. The table shows, for example, that when the monochromator is set for 2537A, the 2482A line is present with energy 2/1000 the energy of the 2537A line.

Relative Energy Values of Spectral Lines

Setting	2399A	2482A	2537A	2653A	3131A	3650A	4358A
2537A	9×10^{-4}	2×10^{-3}	1		7×10^{-4}	5×10^{-4}	7×10^{-4}
2653A	4×10^{-4}	4×10^{-4}	6×10^{-3}	1	4×10^{-4}	3×10^{-4}	5×10^{-4}
3131A	10^{-5}		10^{-4}		1	9×10^{-5}	10^{-4}
3650A	10^{-5}		2×10^{-4}		2×10^{-4}	1	10^{-4}

It was assumed that the amount of infrared light diffusely reflected inside the instrument was nil, because no detectable energy was transmitted through the instrument when the wave-length drum was set for the extreme ultraviolet.

Reflection inside the instrument then seems also to be negligible, and for the limit of accuracy desired in this work, the monochromated beam can be assumed to consist of the wave-lengths of the group corresponding to the monochromator setting.

MICROSCOPIC EXAMINATION

After exposure in the monochromator, the films furnishing the silver bromide grains were developed with freshly mixed amidol developer at room temperature (about 23°) for three minutes, and fixed, both operations being carried out in absolute darkness.[12] The developer has the property[13] that it completely develops exposed grains in a specially coated plate containing but one layer of grains, in three minutes at 18° although unexposed grains are not affected in five minutes.

In the films used in this work, which were of ordinary commercial emulsion, there were, of course, several layers of grains, but control exposures on unexposed films showed that for the same time of development given the exposed films, about 20 per cent of the unexposed grains were also developed, so it was assumed that two minutes longer development at 5° higher temperature than the length of development and temperature at which the one-layer grain plates were developed would give a sufficient approximation to complete development of the films here used.

In order to determine the number of developed grains present per unit area of the film, the central portion of the film exposed to the light

[12] An alternative method which is widely used and which gives the statistical grain size—frequency distribution, is to coat a glass slide with a very thin layer of emulsion and then after exposure, to remove the developed grains by chemical reduction, and examine the undeveloped grains microscopically.

[13] Slade and Higson, Proc. Roy. Soc. *98A*, p. 154; 1920.

was examined under the microscope at a magnification of 1500 diameters. This magnification considerably exceeds the value (900) which marks the limit of resolution of a microscope.[14] A Pointolite-lamp furnished the illumination for the microscope and the condenser, and the remainder of the optical system consisted of a 1/12 in. Reichert N. A. 1.30 oil immersion objective with 12.5 eyepiece provided with an eyepiece micrometer ruled in 1/2 mm squares. One division (1/2 mm) on the scale of the eyepiece micrometer represented 7.7μ on the stage of the microscope.

A count of the blackened grains was made by first centering the microscope upon the exposed portion of the film, and counting the grains of silver in one of the squares of the eyepiece micrometer by slightly altering the focus of the microscope, thus permitting the grains in the different layers of the gelatine to be counted. If a grain touched the "north" or "east" side of a square, it was included in the count, if it touched the "south" or "west" side of a square it was excluded from the count of that square. This process was repeated until 25 squares had been counted.

The average of the number of silver grains in these squares was assumed to represent the gross number of silver bromide grains which had been made developable. In order to obtain some idea of the homogeneity of the emulsion and the errors of counting, the probable error of the count was also calculated.

This gross value of the number of grains per square does not correctly represent the number of grains which has been made developable by the action of the light, because some grains are developable even though the film be not exposed to light. The gross number of grains made developable minus the "fog reading," was regarded as the net number of grains made developable by light action. This fog correction was made by counting the grains in a portion of the film which had been protected from the radiation by a small bit of black paper slipped in front of one corner of the film. One hundred reticule squares were measured for this "fog reading," and the probable error of the mean found.

As some of the squares contained no fogged grains, while the remaining squares contained only a few grains, it would be unfair to calculate the probable error of the mean, from the deviations of the number of grains in each of the hundred squares from the mean of the hundred squares, because the unit of area has been chosen so small.

[14] Drude, loc. cit. p. 98.

For this statistical consideration, a grouping was made in units of 25 adjacent squares. The 100 squares were thus divided up into 36 partitions of 25 squares each,[15] and the averages and the probable errors computed upon this basis.

Knowing the total time of exposure of the film to radiation, the energy density of radiation, and the number of developed silver grains present per mm², it is possible to calculate the ratio of light-quanta incident on the film to the number of developable grains produced.

It will be shown in the Appendix that a commercial silver bromide emulsion reflects approximately 7 per cent of the incident light-energy between the wave-length limits used in this work, and absorbs practically all the ultraviolet radiation falling upon it. If we now assume that each originally light-sensitive silver bromide grain appears on the developed film as a black grain of silver, we can compute the average quantity of incident ultraviolet light energy required to render developable a grain of photographic silver bromide.[16] The absolute quantity of ultraviolet light energy absorbed to render a grain developable will be about 7 per cent less than the incident energy.

[15] By ordinary methods it would take 900 additions to obtain the number of grains in the 36 partitions, but the following artifice reduces the number of operations to 224:

1. Set down the number of grains in each of the 100 squares in a square array, and let the term in the rth row and the sth column be designated by

$$r,s, \qquad r=0,1,\ldots,9; s=0,1,\ldots,9.$$

2. Form the array whose general term is

$$[r,s]$$

by the rule:

$$[r,0]=\sum_{p=0}^{p=4} p,r. \qquad r=0,1,\ldots,9.$$

$$[r,s]=[r,s-1] \quad - \quad s-1,r \; + \; s+4,r. \qquad s=1,2,\ldots,5.$$

A check can be made as follows:

$$[r,6]=\sum_{s=5}^{s=9} s,r.$$

3. The number of grains in each of the 36 different partitions is given by:

$$(0,s)=\sum_{p=0}^{p=5}[p,s] \qquad r=1,2,\ldots,5.$$

$$(r,s)=(r-1,s) \quad - \quad [r-1,s] \; + \; [r+4,s]. \qquad s=0,1,\ldots,5.$$

The check is:

$$(6,r)=\sum_{p=5}^{p=9}[p,r].$$

[16] No attempt was made to determine the frequency of occurrence of the various sizes of silver bromide grains present in the emulsion of the unexposed film. However the *mean* grain-size (assuming that this value has some slight significance) as measured by the diffraction haloes produced by a small aperture was of the order of 0.8μ.

RESULTS

The results obtained from 98 exposures and sets of counts may be summarized in the following table:

INCIDENT ENERGY PER DEVELOPABLE GRAIN OF SILVER BROMIDE

Wave Length	Average Energy. Quanta per μ^2	Average Energy Flux. Quanta per μ^2 per sec.	Exposure Seconds.	Quanta per Grain	Average Quanta per Grain	General Average. Quanta per Grain
	179	12.4	14.4	797±21		
	179	27.9	6.4	780±20	770±14	
	179	49.6	3.6	745±30		
2537A						731±14
	82	11.4	7.2	655±30		
	82	25.8	3.2	700±38	681±25	
	45	12.6	3.6	562±33	562±33	
	50	14	3.6	503±13		
2653A	28	14	2	542±49	514±11	514±11
	72	18	4	584± 8		
3131A	36	18	2	402±14	510±16	510±16
	68	17	4	613± 7	555±13	555±13
3650A	34	17	2	441±13		

The probable errors were calculated only to show the relative precision of measurement between the various means. The absolute values of the means may show greater deviations than the probable errors would indicate due to conditions such as the matter of fog correction, which were taken into account during the experiment but for which some small outstanding correction should be made.

The results show clearly that within the limits of exposures given to the films, small energies are more efficient in producing developable grains than large energies, a fact which may be explained [17] in terms of present theories by saying that if a grain or a "developable center" is struck several times by light-quanta, it behaves exactly in the same way as a grain or a "center" struck but once.

[17] Cf. Silberstein, Phil. Mag. *44*, p. 257; 1922.
Svedberg, Nature, *109*, p. 221; 1922.
Noddack, Streuber and Scheffers, loc. cit.

APPENDIX

THE REFLECTING AND THE ABSORBING POWER OF A PHOTOGRAPHIC EMULSION

In order to determine by the foregoing method the number of quanta of ultraviolet light required per developable grain of silver bromide it is of great importance to know the optical behavior in the ultraviolet of a photographic emulsion and of its constituents.

APPARATUS

The source of the various wave-lengths used in the reflection and absorption measurements was a copper arc and a Hilger large quartz spectrograph which took a 4 in. by 10 in. (10.2 cm by 25.4 cm) plate and gave a dispersion of 18.9 cm between the lines 2015A and 5782A. The lines of the arc were identified with the aid of the table of copper wave-lengths by Hasbach.[18]

At first as Fig. 2 shows, considerable difficulty was encountered in recording wave-lengths shorter than 2369A upon either "panchromatic" or "ordinary" plates. On the basis of the results of Duclaux and Jeantet[19] and of Henri,[20] it was thought that a 1 to 500 solution of lubricating oil in xylene might act as an efficient sensitizer, but a trial showed that the brand of lubricating oil which was at hand had but little sensitizing action at this dilution. A thin coating of the oil applied directly to the emulsion of the plate gave a marked sensitization between the wave-lengths 2369A and 2104A. Before developing the plate however, it was necessary to remove this oil with xylene in order to allow the developer to penetrate into the emulsion. It was later found that a solution of anthracene in carbon disulphide forms as efficient a sensitizer as the lubricating oil and possesses the additional advantage that the sensitizer need not be removed from the plate before development.[21]

THE REFLECTING POWER OF SILVER BROMIDE EMULSION

To obtain an approximate measure of the reflecting power of commercial silver bromide emulsion in the range 2104A to 5782A it was desired to obtain spectrograms representing three intensities of the copper arc: unit intensity, 1/10 unit intensity, and 1/100 unit intensity; and then to obtain a fourth spectrogram upon the same plate by allowing unit

[18] Zeit. wiss. Phot. *13*, p. 402; 1914.
[19] Journ. Physique et Radium. *2*, p. 156; 1921.
[20] Journ. Physique et Radium. *3*, p. 181; 1922.
[21] Cf. also Luckiesh, *Ultra Violet Radiation*, p. 199. Van Nostrand. 1922.
Lyman, Nature. *112*, p. 202; 1923.

intensity from the copper arc to be reflected from the emulsion surface whose reflection coefficient was to be found. If for example the intensity of the copper lines in this fourth spectrogram was the same as in the spectrogram obtained by direct exposure to 1/10 unit intensity of the arc, the reflecting power of the emulsion would be 10 per cent. Visual

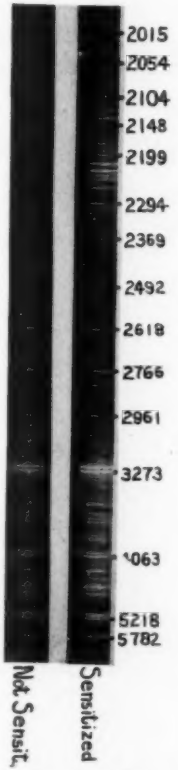

FIG. 2. *The sensitizing action of ordinary lubricating oil.*

comparisons of the intensities of the different spectrograms would thus give a rough determination of the reflecting power of silver bromide emulsion.

This plan was carried out by setting up the copper arc 20 cm. from the slit of the spectrograph and placing a square plate of fused quartz (3 cm by 3 cm by 2 mm) with carborundum-ground surfaces 5.5 cm

from the arc, to act as a diffusing screen. A diaphragm with a circular aperture of 25.40 mm diameter was placed in the "parallel" beam of light between the prism and the collimator lens farthest removed from the slit of the spectrograph, and a spectrogram exposed for 3 minutes with the arc fed with constant energy input of 350 watts.

Another spectrogram was then exposed for the same length of time and with the same arc conditions, except that a diaphragm with circular aperture of 7.91 mm was substituted for the diaphragm of larger aperture. A third exposure was similarly made with a 2.54 mm diaphragm. Thus the three spectrograms on the plate represented exposures for the same length of time, but with intensities (assuming no aberrations in the lens) varying in the ratio: 1, 1/10.3, and 1/100.

Next a transparent crystal quartz plate (3 cm by 3 cm by 2 mm) was coated with a photographic emulsion of about the same thickness as found on an ordinary film, by dissolving the emulsion from 9 cm^2 of a piece of commercial photographic film in a few cubic centimeters of warm distilled water, and then pouring the entire solution upon the quartz plate and allowing it to dry. The whole process was performed with light from a ruby safe-light, and the prepared plate was protected from the light until it was ready to be used.

This prepared plate was then oriented with an angle of incidence of 18° and placed about 7 cm from the diffusing screen, so that it reflected light from the arc and diffusing screen upon the slit of the spectrograph. With a light path of the same length as in the three preceding exposures, a spectrogram of the emulsion-reflected light was taken with the largest diaphragm inserted in the spectrograph.

Examination of several sets of plates as in Fig. 3 indicates a value of the reflecting power of a photographic emulsion which ranges between 6 per cent and 8.5 per cent for the region 2100A to 5200A. Measurements on the absorption of light by an emulsion show that there is no appreciable error entering into the above figures due to failure to take into consideration the results of reflections at the emulsion-quartz surface and the quartz-air surface. However the figures given above may be considered to be minimum values, depending upon how far short the diffuse light from the double-ground quartz plate departed from absolute diffusivity, and the fact that due to the distance of 7 cm between the diffusing screen and the reflecting surface, the light incident upon the reflecting surface was not absolutely diffuse.

Spectrograph shown in Fig. 4 were made in a similar way to show the reflecting power of a thin layer of Nelson's No. 1 Photographic Gelatine

coated on quartz. The reflecting power of the entire combination is practically constant and equal to about 10 per cent throughout the whole spectrum 2200A to 5780A.

As a test for possible constant errors in the apparatus caused by some unknown source, spectrograms were made showing the reflection from a 2 mm plane-parallel crystal quartz plate cut perpendicularly to the

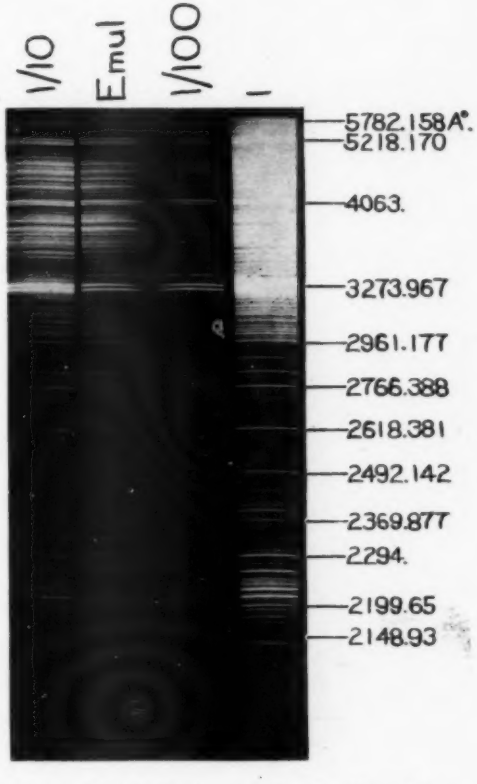

FIG. 3. *The reflecting power of silver bromide emulsion.*

optic axis. (See Fig. 5). With an angle of incidence of 18° the total amount of light reflected from this transparent quartz plate was about 10 per cent for all wave-lengths.

Approximate calculation shows that this value of the reflecting power is of the right order of magnitude.

In order to avoid a complicated calculation[22] in estimating the reflecting power of a plate of quartz, it seems an allowable approximation to assume that quartz is an isotropic substance with but one index of refraction,—that of the ordinary ray,—since the two refractive indices of quartz differ from one another by only 0.74 per cent at 2140A and by

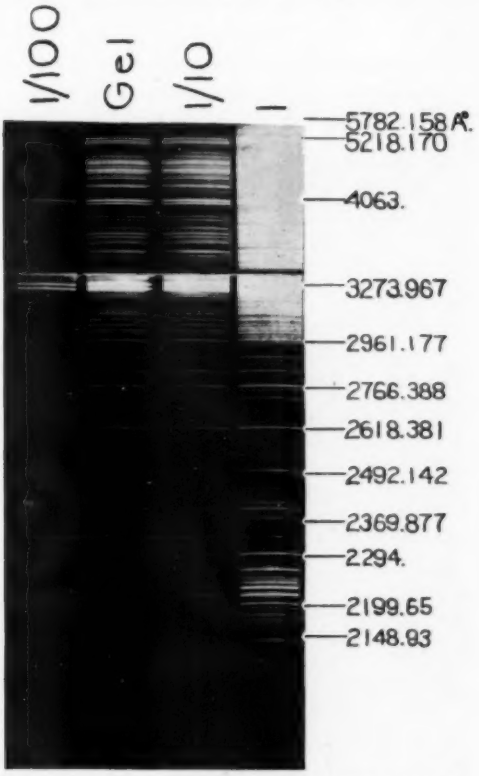

FIG. 4. *The reflecting power of a gelatine-quartz combination.*

0.58 per cent at 5890A.[23] It will also be assumed for the purpose of calculation that within this range of wave-lengths the 2 mm quartz plate is perfectly transparent. The work of Coblentz[24] shows that quartz is practically completely transparent from 2500A to 17000A.

[22] Cf. Drude, Lehrbuch d. Optik, p. 319; 1900.

[23] Gifford. Proc. Roy. Soc. *70A*, p. 329; 1902.
Cf. also a similar assumption made by Coblentz. Bull. Bur. Standards. *11*, p. 471; 1915.

[24] Loc. cit.

After calculating from the Fresnel formula the reflecting power (R) of an air-quartz layer for an angle of incidence of 18°, the reflecting power of the quartz plate is easily found from the expression $2R/(1+R)$.[25] For wave-length 2140A the reflecting power is found to be 12.22 per

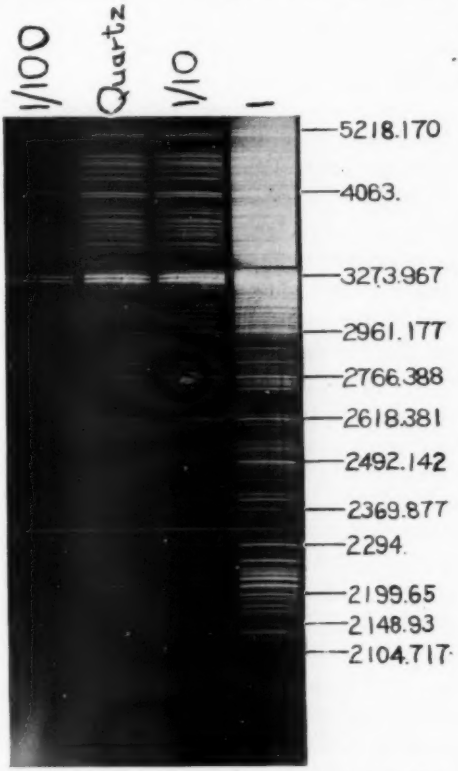

FIG. 5. *The reflecting power of a transparent quartz plate.*

cent, and for 5890A the reflecting power of the 2 mm quartz plate decreases to 9.61 per cent.[26] Comparison of these figures with the observed

[25] Winkelmann. Nandb. d. Physik. *6*, p. 1252; 1906.

[26] If the absorption of quartz be considered, (at 2140A, 9 mm of quartz absorbs 8 per cent,—Pflüger, Physikal. Zeit. *5*, p. 215; 1904) the value of the reflecting power at 2140A should be decreased by 1.7 per cent. Furthermore, it is interesting to notice that the assumption of normal incidence instead of an angle of incidence of 18° would not have appreciably changed the calculated values of the reflecting power, for the resulting values with normal incidence are 12.17 per cent, and 9.58 per cent. Since the gelatine which envelops the silver bromide grains must exert a powerful influence upon the phenomena of reflection, the fact that the index of

value (about 10 per cent) shows the relative accuracy which may be expected by the use of this method.

THE ABSORPTION OF THE SILVER BROMIDE AND THE GELATINE IN AN ORDINARY PHOTOGRAPHIC EMULSION

The method used to obtain a measure of the absorption of the silver bromide found in an ordinary film was somewhat similar to the method employed in obtaining reflection coefficients.

As before, a quartz plate was coated with the same weight of emulsion per cm^2 which exists in an ordinary photographic film. The emulsion of the films used for this work measured 0.021 mm in thickness; and the average weight of the emulsion of these films per cm^2 averaged 4.2 mgm, the silver bromide content of the emulsion weighed 2.4 mgm/cm^2, the gelatine component 1.8 mgm/cm^2.

With the coated quartz plate placed before the slit of the spectrograph together with the ground-quartz diffusing screen in position, and with the largest diaphragm in place in the spectrograph, a spectrogram was made of the copper arc. The arc stood 18 cm in front of the slit of the spectrograph, with the ground-quartz screen 10 cm from the arc and the emulsion-coated side of the transparent quartz plate was placed against the ground-quartz surface nearest the slit of the spectrograph.[27]

As only the absorption of the silver bromide was to be considered in these measurements, the effect of the gelatine in the emulsion was neutralized by placing between the arc and the spectrograph a quartz plate coated with the same quantity of gelatine found in an equivalent area of photographic film. The "unit," "1/10" and "1/100" comparison exposures were made with this compensating plate in place. These comparison exposures actually represented relative intensities of 0.97, 0.097, and 0.0097, because the respective reflecting powers of gelatine and of photographic emulsion are about 10 per cent and 7 per cent.

Examination of the plates (Fig. 6) showed that the silver bromide of the emulsion transmitted less than 1 per cent[28] of the radiation of

refraction of photographic gelatine for the *D* line differs from the index of the ordinary ray of quartz by less than 1 per cent (index of gelatine 1.530, index for the ordinary ray through quartz 1.544,—Smithsonian Physical Tables, pp. 281, 283; 1920) seems to indicate that the value of the reflection coefficient of silver bromide for normal incidence would probably not differ greatly from its value for an angle of incidence of 18°.

[27] This method of measuring the absorption of turbid media by placing the material to be measured against a diffusing screen is recommended by Callier, Ferguson (Phot. Journ. *52*, p. 283; 1912), Bloch and Renwick (Phot. Journ. *56*, p. 49; 1916), et. al.

[28] At 4063A about 92 per cent of the *incident* radiation would be absorbed. Weigert (Zeit. Phys. Chem. *99*, p. 499, 1921) finds that at this wave-length the absorption of a photographic emulsion may amount to more than 84 per cent.

wave-length less than 4063A which penetrated the surface of the emulsion, and less than 0.01 per cent of the radiation whose wave-length was less than 3247A. It is thus seen that the silver bromide content of an ordinary film can be considered to be practically opaque to ultraviolet light.

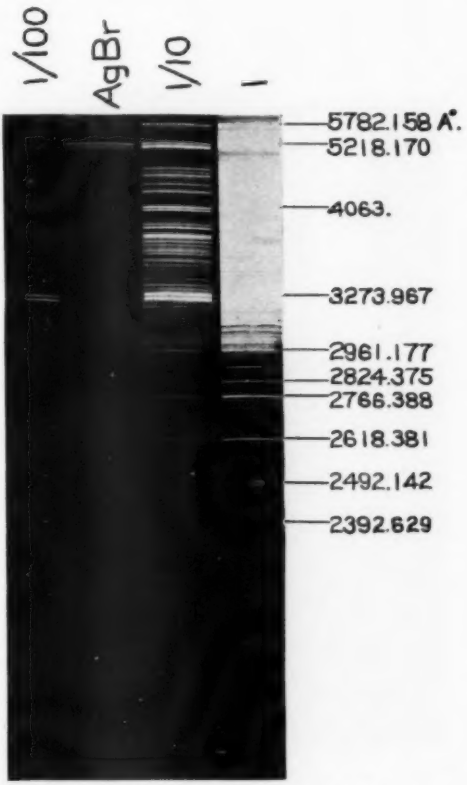

FIG. 6. *The absorption of the silver bromide in an emulsion of ordinary thickness.*

Measurements were also made on the absorption of the gelatine content of an ordinary photographic film. A piece of film 9 cm^2 in area was thoroughly fixed and washed, and the resulting gelatine content was coated on a 3 cm by 3 cm quartz plate which was then placed against the diffusing screen, and a spectrogram taken. The gelatine-coated quartz plate was then removed and a quartz plate of equal

thickness but having no gelative coating was substituted, and the three comparison exposures impressed upon the plate, Fig. 7.

Previously mentioned measurements have shown that quartz and gelatine have practically the same coefficients of reflection for all wavelengths, and because the refractive indices of quartz and gelatine for

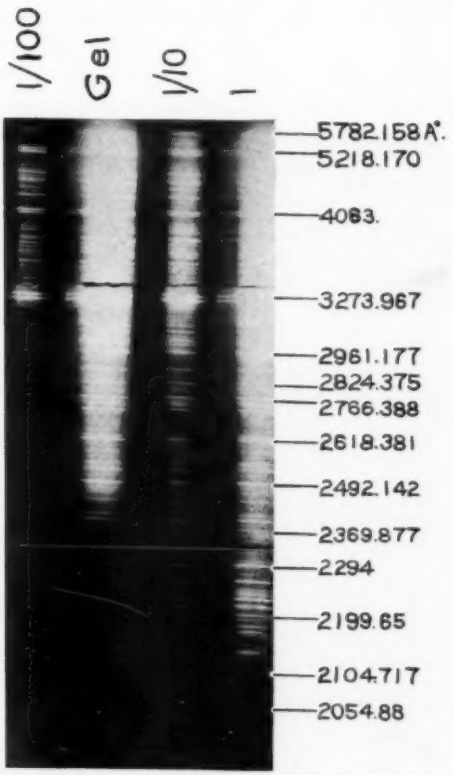

FIG. 7. *The absorption of the gelatine in an ordinary photographic film.*

the *D* line differ from each other by only a few per cent,[29] it may be assumed that the reflection at the quartz-gelatine surface may be neglected, consequently the spectrogram represents the true absorbing power of the gelatine layers.

The original negatives show that practically no light more refrangible than 2369A is transmitted by gelatine, but that at this wave-

[29] Smithsonian Physical Tables, pp. 281, 283; 1920.

length about 20 per cent is transmitted. For wave-lengths longer than 2441A, there is no appreciable absorption.

Schumann[30] has found that a 0.02 mm layer of Heinrich's Hard Emulsion Gelatine begins to absorb at 3253A, and reaches a maximum at 2307A. For a layer 0.01 mm thick, the first noticeable absorption

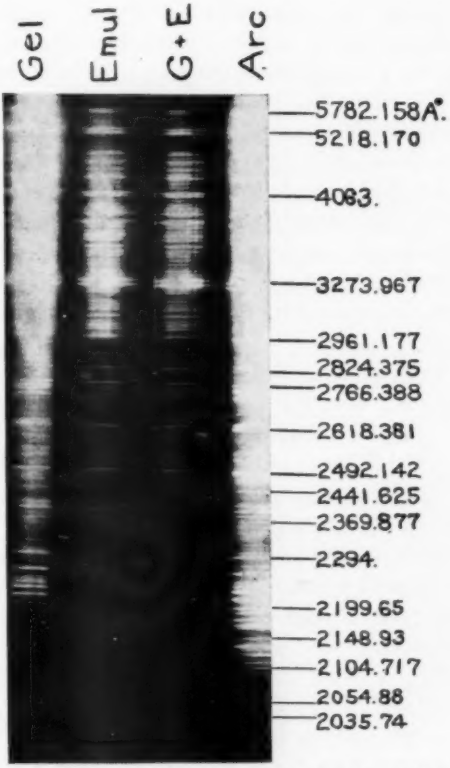

FIG. 8. *The relative absorptions of an emulsion and its gelatine content.*

begins at 2880A, and the maximum is attained at 2307A. As the gelatine contained in an ordinary emulsion is equivalent to a layer about 0.014 mm thick[31] the writer's values should lie within the two sets of values given by Schumann. There is fair agreement on the short wave-length side.

[30] Sitz. Wiss. Wien. *102*, IIA, p. 459; 1893.

[31] Assuming that gelatine has a density of 1.27 gm/cm³ (Smithsonian Tables, p. 113).

Finally in Fig. 8, the absorptions of an emulsion and of its gelatine content are shown. For this purpose a quartz plate inclined at a slight angle with the horizontal was coated with emulsion so that when dry a wedge of emulsion with angle of about 4′ was formed on the quartz. A similar wedge of gelatine was prepared with an equal volume of emulsion from which the silver bromide had been removed by thorough fixing and washing. Spectrograms were made with the respective wedges in front of the arc singly, and in combination to compensate for the absorption of the gelatine when the absorption of the silver bromide alone in the silver bromide emulsion was being studied. No diffusing screen was used in this case, because of the extremely small amount of silver bromide present in the thinnest portion of the wedge, nor has any allowance been made for reflection in the case of the spectrogram representing the absorption of the silver bromide content of the emulsion in the superposed wedge spectrogram. The plates showed that up to at least 2294A, a certain thickness of gelatine has a negligible absorption for the near ultraviolet when compared with an equal thickness of emulsion.

The writer takes pleasure in acknowledging the loan of a microscope and of an objective from the Departments of Geology and of Biology of Princeton University, and of a Hilger Illuminator from Professor W. M. Clark of the Hygenic Laboratory, Washington, D.C. His thanks are also due the Director and staff of Palmer Physical Laboratory of Princeton University where this work was carried out, for their kindness in placing the facilities of the Laboratory at his disposal. In conclusion, he wishes to express his gratitude to the National Research Council for the grant of a fellowship which made this work possible.

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Note on Astronomical Objectives of Two Cemented Components.—The author finds that the additional condition for the suppression of coma in an astronomical objective is equivalent to rendering equal the roots of the quadratic equation in the Abbe zero-invariant expressing the conditions for achromatism and freedom from spherical aberration at the rear focus. This is true even in the Harting objectives where the constituent glasses have nearly the same average indices of refraction but different dispersions. [M. A. Biot, Annales de la Société scientifique de Bruxelles, *43*-I, p. 92; 1923.] P. MERTZ

VISIBLE RADIATION FROM SOLID TARGETS[1]

By Paul D. Foote, W. F. Meggers and R. L. Chenault

It would not be surprising if it were possible to excite a visible or ultraviolet line spectrum by bombarding solid targets of certain metals with electrons. According to Bohr's theory, the valence electron of an alkali atom moves for the greater part of its path in an approximately hydrogenic field on an elliptical orbit, the dimensions of which are determined by the effective quantum number n^*, as obtained from the proper spectroscopic term, and the corresponding azimuthal quantum number k. The aphelion distances so derived are larger than the shortest distances between centers of adjacent atoms as determined by x-ray crystal analysis. However, in copper there is a d term of high wave number, supposedly due to a deep lying 3_3 orbit and Bohr attributes the $2p$ term to a 2_2 orbit having dimensions not materially greater than the space lattice. There is accordingly a possibility that a spectral line could be excited in copper although in this particular case, we should expect considerable modification of its frequency by the fields of adjacent atoms since even the 2_2 optical orbit is unfavorably large. However, it may be that some of the metals have a space lattice so large that a few optical orbits can exist without serious perturbation, so that optical lines or at least diffuse bands could be excited.

A second and far more promising possibility lies in the fact that on the Moseley diagram some of the lines representing x-ray limits or energy levels cross one another in the range of lower frequencies. The selection principle for the third quantum number, the ab rule of Bohr and Coster, is an empirical relation developed only for shells having their full complement of electrons. We do not know how this rule applies to atoms where an underlying shell is undergoing a stage in development. If it does not hold under such conditions, numerous x-ray lines of very small frequency, lying even in the visible spectrum, may arise from electronic disturbances fairly deep down in the atom, produced by transitions between levels on the Moseley diagrams near the points of their intersection. These should be sharp lines since the perturbing effects due to adjacent atoms should be very slight. Even if the selection principle holds, sharp lines may be expected in the far ultraviolet—within reach of the vacuum spectrograph. It may be

[1] Published by permission of the Director of the Bureau of Standards of the U. S. Department of Commerce.

that the almost monochromatic, fluorescent spectra of some of the rare earths are due to interorbital transitions of the above described character.

Accordingly, we have photographed the spectra obtained when copper, carbon, platinum, and iron targets were bombarded with 1000-volt electrons. Every precaution was observed in the matter of vacuum technique. Diffusion pumps were operated for several days before making an exposure; the apparatus was thoroughly baked and the metal parts and target were heated for hours to a temperature where perceptible vaporization occurred; double liquid air traps were employed, one of which was baked. The x-ray bulb was of pyrex glass with a quartz window sealed in by the use of intermediary glasses.

In spite of these precautions the same group of lines appeared, for a current of several milliamperes and 5 or 6 hours exposure, with all the targets. Some of the more prominent bands and their relative intensities are as follows:

Line	Intensity	Line	Intensity
2897.1	10	2446.6	10
2883.8	10	2420.2	10
2607.8	3	2353.1	2
2578.3	7	2325.8	8
2551.1	4	2300.3	10
2475.0	5	2190.5	5

These bands are listed in Kayser's Handbook V, p. 233 as the first negative carbon group of Deslandres who discovered them near the cathode in discharges below 1/20 mm pressure in CO, CO_2 and C_2H_2. We have found however, that they are enhanced by the presence of nitrogen. For example, if nitrogen[2] is admitted to the tube at a pressure of 0.005 mm, the bombardment at 1000 volts gives the second positive band group and the negative pole bands of nitrogen while nitrogen lines are absent. Below 0.002 mm, the ordinary band spectra are almost totally replaced by a complicated spectrum in which the lines listed above are prominent and become relatively more prominent as the pressure is further decreased. It is, of course well known that CO is occluded in metals and glass but whether these lines are due to CO

[2] This work was done in collaboration with Dr. Arthur E. Ruark, in an attempt to excite the green aurora line with high voltage electrons in nitrogen at very low pressures. The aurora line is not present in spectra obtained at 1160 volts with a Hilger E_2 spectrograph.

or to N, their excitation at the extremely low pressure at which our tubes were operated is quite surprising. Our experiments are therefore negative in demonstrating an emission of visible or ultraviolet spectral lines by solid targets and the extreme difficulty in obtaining conclusive evidence of this nature is clearly indicated.

A definite continuous spectrum extending from the red to the extreme ultraviolet limit of the plate was observed with all targets. In general, it appeared very faint for wave lengths shorter than 2500 A on account of the rapid decrease in plate sensitivity in this spectral range. This is undoubtedly general x-radiation. Lilienfeld[3] has made similar observations in the visible spectrum.

If the electron current is such that the target becomes heated, an almost imperceptible volatilization is sufficient to excite the resonance lines of the element. For example, when the copper target is heated to 900° C, at which temperature the vapor pressure is of the order 10^{-4} mm Hg, the lines $1s-2p$ are the strongest on the plate and are still intense at very much lower pressures. The iron target contained a slight trace of carbon and manganese. The carbon lines 2478 (5) and 2512 (1) and the manganese triplet, $1S-2P$, 2794 (3), 2798 (3), 2801 (3) were observed. It appears probable that these lines are produced by the photoelectric action or resonance scattering of the general x-radiation from the target rather than by direct electronic impact.

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WASHINGTON, D. C.

Consecutive Appearance of various Arc Lines of Mercury at potentials below ionization.—In these observations the tube is so shaped and the electrodes so arranged that light emitted from mercury atoms very near the anode of the apparatus reaches the spectrograph after a very short passage through mercury vapor: it is suspected that in ordinary arrangements the impinging electrons fail, owing to space-charge, to approach the full energy corresponding to the anode-cathode P. D. until they arrive quite close to the anode. Photographs are reproduced which were taken at various voltages between 7.7 and 10.4 the exposures lasting several hours. Below 7.7 only the line 2536 ($1S-2p_2$) is visible; but the voltage 7.9 should just suffice to bring the valence electron from the $1S$ orbit into either the $2S$ or the $2s$ orbit, from which there are five possible transitions back into lower energy orbits; the photograph taken at 8.4 volts shows 4 of these lines, the other one being in the inaccessible red. A voltage of 8.8 should just suffice to bring the electron into the $3D$ and $3d$ orbits whence 11 new transitions

[3] Lilienfeld, Phys. ZS. *20*, pp. 280-2, 1919; idem *21*, pp. 249-56, 1920.

are possible; eight of these, falling into three compact groups, correspond to lines in the accessible range, and these groups are observed (and partially resolved) in a photograph taken at 8.9 volts. Further new lines appear at voltages between 9.6 and 10.4 as predicted. This appears to be the most definite contribution yet made to the solution of the question whether or not there are "selection principles" inhibiting the lifting of the valence electron by electron impacts from its stable orbit to some of the transient-sojourn orbits. [J. A. Eldridge, University of Wisconsin; Phys. Rev. (2) *23*, pp. 685-692; 1924.]

K. K. DARROW

Method for the Determination and Measurement of the Aberrations of Optical Systems.—A line grating having from 10 to 20 lines per millimeter is placed near the focal plane of the objective whose characteristics are to be determined. By means of a diagonal transparent mirror monochromatic illumination is provided in such a direction that the light proceeds through the grating to the objective. A plane mirror is placed in front of the objective in such a position that the illuminated portion of the grating coincides with its image by autocollimation. The observer's eye, placed in the returning beam back of the diagonal transparent mirror, views the objective and sees the surface traversed by black lines that appear to be localized on the lens. Displacement of the eye perpendicular to the axis of the system does not cause movement of the dark bands with reference to the objective. If the eye approaches the grating the dark lines do not undergo any change.

With an objective well corrected for spherical aberration the lines are straight, parallel, and equidistant. The distance between the lines increases when the grating is moved toward the focal plane and the lines disappear when the grating and the focal plane are coincident.

But if the lens is poorly corrected for the geometrical aberrations the dark lines have a complex form that is variable according to the position of the grating.

Observation of these various aspects enables one to determine the position of the focal plane of a good objective as well as to study and measure the aberrations of lenses not so well corrected.

A specially constructed viewing device is used that places a pupil in the grating plane. Photographic records may be made by introducing a camera.

To observe the chromatic aberration a slit is placed between the objective and the plane mirror, the slit being perpendicular to the lines on the grating, and a direct vision prism introduced into the viewing system. The spectrum obtained is crossed by dark bands, from the location and form of which the numerical values of the chromatic error may be deduced.

The adjustment of the apparatus is easy and does not require specially trained observers. [M. L. Lenouvel, Revue d'Optique, *3*, pp. 211-243; 1924].

G. W. MOFFITT

THE INFRARED ABSORPTION OF ICE

By E. K. Plyler

SYNOPSIS

The absorption of ice is dependent upon the direction of the plane of polarization for the near infrared region. Crystals were cut with the optic axis parallel to the surface. Plane polarized radiation was allowed to fall upon the crystal and the per cent transmission determined. The crystal was set in three positions: first, so that the transmitted radiation constituted the ordinary ray, second, so that the transmitted radiation constituted the extraordinary ray, third, so that the incident energy was about equally divided into two rays. For the first position, with a thickness of 5mm, absorption bands were observed at the following wave-lengths with the corresponding per cent transmission: .81μ—55 per cent, .92μ—54 per cent 1.06μ—46 per cent, 1.29μ—27 per cent. For the second position the values were: .79μ—93 per cent, .89μ—94 per cent, 1.02μ—93 per cent, 1.26μ—57 per cent. For the third position the transmission curve fell between that of the first and second positions and showed both sets of bands. The region from 1.4μ to 1.7μ was studied by using a specimen 1 mm thick. The position of maximum absorption was located at 1.5μ. The incident energy was unpolarized.

I. Introduction

Since Aschkinass[1] first measured the spectral transmission of water in the infra-red, there have been very few attempts to measure the spectral transmission of ice. We should expect that the infrared absorption of water and of ice should be similar as each substance has a dielectric constant of about 80 for long waves. Water has an index of refraction slightly larger than the index of refraction of ice, and its dispersive power is different. This fact indicates that there is some difference in their spectral transmission in the infrared. Hettner[2] has measured the absorption of water vapor in the infrared. He found that water vapor had an absorption spectrum similar to that for water, but that each absorption band of water vapor was located on the short wave-length side of the corresponding absorption band of water. When the absorption spectrum of water and water vapor are compared for equal molecular thicknesses, the intensities are unequal.

Numerous measurements of the spectral transmissions of liquid water have been made at room temperature by different investigators. However, Collins[3] has recently studied the absorption of water at various temperatures. He found the two following changes in the absorption spectrum of water: first, the intensity of the absorption bands varied with temperature; second, the position of maximum absorption

[1] Annalen d. Physik, *55*, p. 401; 1895.
[2] Annalen d. Physik, *55*, p. 476; 1917.
[3] To be published.

shifted toward the short wave lengths at higher temperatures and toward the long wave lengths at lower temperatures. It is of interest to compare the absorption spectrum of ice with that of water at 0°C.

Sutherland[4] suggests that water is a mixture of $(H_2O)_2$ and $(H_2O)_3$, and that ice is $(H_2O)_3$. If this is correct, we would expect ice to have a different absorption spectrum from that of water in the infrared region.

Saunders[5] was the first investigator to study the infrared absorption spectrum of ice. He made a cell by separating sheets of mica with a thin ring. The cell was filled with water and then placed in a freezing mixture. The absorption of ice was determined for the region from the visible spectrum to 3.2 μ. Two curves were determined for thicknesses of .025 cm and .08 cm. By allowing the ice to melt the absorption of a layer of water of equal thickness was determined. Saunders came to the conclusion that the absorption of ice and that of water were equal for the region from the visible spectrum to 3.2 μ. In 1909 G. Bode[6] made a study of the infrared absorption of ice. He obtained thin layers of ice by freezing distilled water between glass plates. He found that a layer of ice .01 cm thick absorbed 93 per cent of the radiant energy at 1 μ. The absorption spectrum of ice had an equal number of absorption bands, as found in water, located at the same wave lengths. Bode also found in the region from 1 μ to 2.9 μ that ice had a general absorption. On account of this general absorption the per cent transmission of ice was found to be much less than that of an equal thickness of water for all wave lengths in the region which was investigated.

It was felt that these results did not give correct values for the absorption of ice. The present work was undertaken so that there would be available a set of values of the absorption of ice in the infrared region which could be used for comparison with the absorption of water in the infrared.

II. Apparatus and Experimental Method

The writer was fortunate in having available the apparatus which was used by Collins.[7] In order to obtain a spectrometer system free from appreciable stray radiation two spectrometers were used, one serving as a monochromatic illuminator for the slit of the second. As measurements were to be made to 2.3 μ only, spectrometers with glass

[4] Phil. Mag., *50*, p. 460; 1900.
[5] Johns Hopkins Univ. Circ., No. 140; 1899.
[6] Annalen d. Physik, *30*, p. 326; 1909.
[7] Phys. Rev., N.S., *20*, No. 5, p. 486; 1922.

lenses and prisms were used. The spectrometers were of the constant deviation type.

It was found impossible to obtain clear ice by freezing distilled water between glass plates. By cutting sections out of a 300 lb. block of artificial ice very clear pieces of ice were obtained. The ice was cut to fit a piece of brass tubing and was sealed in it by freezing a layer of distilled water which filled the space between the ice and the tubing. The tubing was 2 cm in diameter and of different lengths. The tubing was accurately cut on a lathe. This method of cutting made the two ends plane parallel. The piece of ice which was frozen in the tubing was cut sufficiently long so that it extended out of both ends. Thus it was possible to polish the ice. When the ice was polished down to the brass edge at both ends, the two surfaces were plane parallel. The ice was polished first with leather, then chamois and last with silk. The polishers were made by stretching several layers of the material over a smooth board. If the ice was kept several degrees below freezing, the polish would be retained on the surface.

A cooling cell was used to keep the ice below the melting point. The cell consisted of two concentric brass tubes about 2.1 cm long. Brass plates had discs cut out of them of a diameter equal to the diameter of the inner tube. Then these plates were sealed on the tubes leaving both ends of the inner tube open. A cooling solution could be introduced through a small hole in the larger tube. The brass tube, in which the ice was mounted, fitted closely into the inner tube of the cooling cell. The ice could be used for one half hour without the surfaces losing their polish.

In order to avoid the error due to the reflection at the surfaces, two specimens of ice of different thicknesses were used. The specimens were placed in the cooling cells. The cooling cells were mounted on a slider which could put either specimen in the beam entering the spectrometer system. The ratio of the deflections of the galvanometer for each specimen in the beam gives the per cent transmission of a layer of ice equal in thickness to the differences in thickness of the two specimens. Two determinations were made of the per cent transmission at each setting of the spectrometer. These values did not ever differ by more than 2 per cent. In many cases the two values differed by a fraction of one per cent. After a complete set of readings was taken, the readings for the first spectrometer setting were repeated to see if any change had occurred.

At first it was not possible to obtain two curves which did not differ considerably from each other. The spectral transmission was first determined in the region of 1 μ. The difference in thickness between the two specimens of ice was 2 cm. In Fig. 1 the transmission curves for two different sets of readings are given. The two curves shown differ in the per cent transmission by 14 per cent in the region of greatest absorption. Also, the most absorbing region of curve A is at 1.02 μ and for curve B at 1.04 μ. The difference in the per cent transmission was not due to experimental errors. Eight sets of observations were

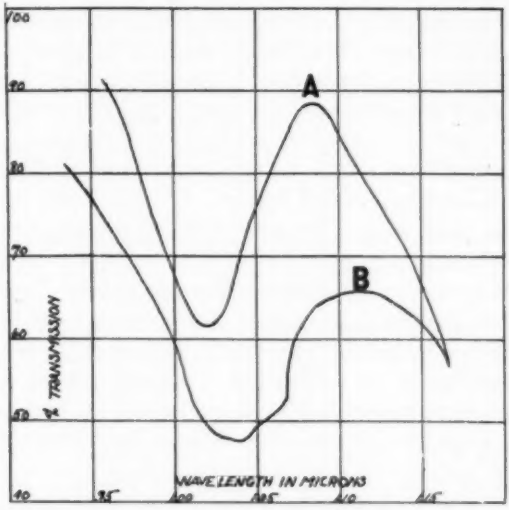

FIG. 1

made for this region with different specimens. However, the transmission curves did not correspond even approximately in any two trials.

After some investigation the cause of the large variation in the spectral transmission for the different specimens was discovered. It was found that the spectral transmission depended upon the direction of the optic axis in the crystals of ice as well as on the thickness. The radiant energy passing through the spectrometer system was partially polarized by reflection at the surfaces of the prisms. By testing with a nicol prism it was observed that about twice as much of the radiant energy was transmitted when the electric vector was horizontal as when it was vertical. Thus if the more absorbing direction in the

crystal was horizontal, the spectral transmission would be less for the specimen of ice than when the more absorbing direction was vertical.

This fact made it necessary to work with single crystals and to have the optic axis in a definite position. It was important to obtain individual crystals of sufficient size to entirely fill the brass tube in which the ice was mounted. Large crystals were obtained by keeping pieces of artificial ice slightly below the freezing point for about 2 months. In this way some crystals over 5 inches long and others over 1 sq. in. in cross section were obtained. The two pictures in Fig. 2 show the appearance of the crystals when the surface had been melted slightly to bring out the lines of separation. The pictures are about ¼ actual size. Geologists have made an extended study of the growth of ice crystals. Large pressures tend to increase the size of the crystals. Also high temperatures (below zero centigrade) favor the growth of the crystals of ice. At present there is no complete explanation as to how the small crystals disappear and the larger ones increase in size.

Ice crystals are positive and belong to the hexagonal system. Meyer[8] has found that the refractive index for the ordinary ray was 1.308, while the value for the extraordinary ray was 1.313, the source being sodium light. The direction of the optic axis was located by the use of a converging polariscope. The optic axis could also be located by observing the direction in which small particles of frost would form on the ice. The formation of the frost on the ice took place each time the temperature of the refrigerator was lowered. Each little crystal added by the frozen vapor has its optic axis parallel to the optic axis in the crystal. In general the optic axes in adjoining crystals are not parallel and thus the small added crystals point in one direction for one crystal and in another direction for another crystal. By observing the reflection of a light from the surface of the ice certain sections appear bright. By changing the position of the eye the former bright sections disappear and other sections become bright. This effect was striking for certain positions as at oblique incidence some sections reflect the light in the direction of the source.

The crystals of ice were cut so that the optic axis was parallel to the polished surfaces. That is, the optic axis was perpendicular to the beam of the incident energy. It was advisable to work with only one specimen of ice at a time and make corrections for the loss of intensity by the reflections at the surfaces. Since the position of the optic axis has a

[8] Smithsonian Physical Tables, 7th Ed.

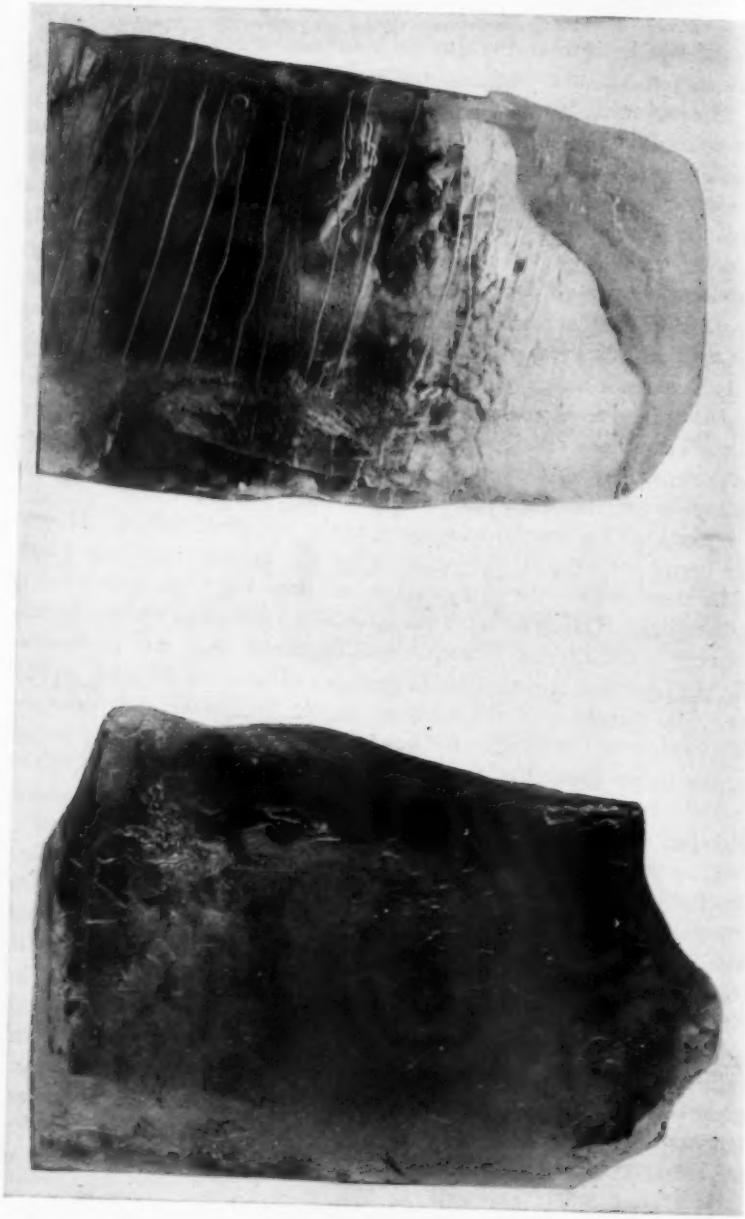

FIG. 2

great influence on the per cent transmission, a slight difference in the direction of the optic axis in two crystals would introduce an appreciable error. In order to obtain a plane polarized beam of radiant energy a nicol prism was placed in front of the specimen of ice. The extraordinary vibration in the nicol constitutes the transmitted energy. For this direction of vibration the calcite is almost transparent for radiation to 3 μ. Crossed nicols would not transmit any radiation in the near infrared region. This showed that by the use of one nicol, the radiation was plane polarized. The electric vector of the Nicol prism was set horizontal so that the greatest intensity would be transmitted through the spectrometer system.

III. EXPERIMENTAL RESULTS

The spectral transmission of a crystal of ice 0.5 cm thick was determined for the following three positions: first, so that the transmitted radiation constituted the extraordinary ray; second, so that the trans-

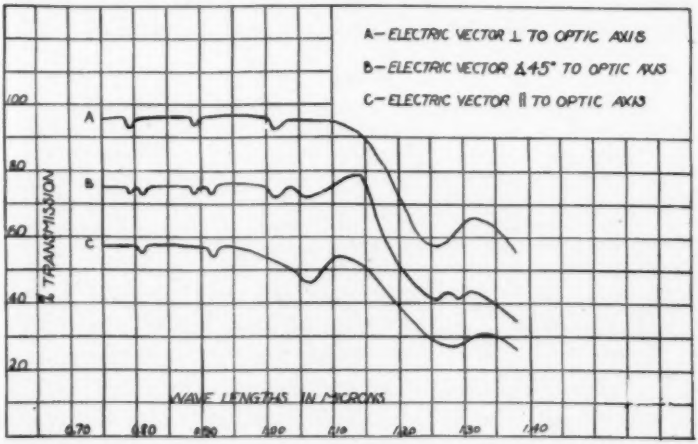

FIG. 3

mitted radiation constituted the ordinary ray; third, so that the incident energy was about equally divided into the ordinary ray and the extraordinary ray. This third position will be referred to as the mixed ray. The results obtained for the three positions are shown in Fig. 3. It is interesting to note that the mixed ray shows both sets of absorption bands, and the per cent absorption is between that of the ordinary ray and the extraordinary ray. The extraordinary ray does not exhibit any general absorption in the region extending from 0.75 μ to 1.15 μ, while

in the ordinary ray there is a general absorption in this region. Each ray has the corresponding absorption bands as found in liquid water, but the bands for the ice are shifted toward the long wave lengths.

The region as shown in Fig. 3 was studied with special care, as many as ten curves with different specimens being made for almost all of it. Readings were taken at intervals of 0.01 μ. In a few cases there was a tendency for all the absorption bands to have their maxima shifted toward the visible region. This shift never amounted to more than .01μ. It is probable that this change was due to the ice being at a higher temperature. In Table 1 are given the values in microns for the location of the absorption bands for the two rays of ice and the values for water at 0°C. These determinations were made on the same instrument. The location of the absorption bands for water have been found by several observers. The values in any two cases do not correspond (difference likely due to calibration). Yet the values given here show the relative positions of the two sets of absorption bands of ice and of water for equivalent thickness (i.e. for equal masses of material per square centimeter of absorbing layer).

TABLE 1.

Thickness of ice .5 cm Thickness of water .458 cm

Ordinary ray		Extraordinary ray		Water at 0°C	
Wave-length	per cent trans.	Wave-length	per cent trans.	Wave-length	per cent trans.
.79μ	93	.81μ	55	.745	98*
.89	93.5	.92	54	.845	98*
1.02	93	1.06	46	.98	93.7
1.26	57	1.29	27	1.215	55.6

* Taken at room temperature.

As only one specimen of ice was used, it was necessary to correct for the loss of intensity due to reflections at the surfaces. The reflection correction was calculated by the use of Fresnel's formula, the value of the index of refraction being taken as that found for the sodium lines. A small error is introduced by the assumption that the reflection is the same in the visible and in the near infrared regions. However, since the correction for reflection at two surfaces was less than 4 per cent, the influence of this error on the result would be very small.

With a layer of ice 0.5 cm in thickness it was difficult to extend the absorption curves beyond 1.4 μ. Thin specimens of ice could be ob-

tained by freezing distilled water between glass plates, but this method does not produce clear ice. The following method was used. A brass ring 1 mm thick was imbedded in a crystal of ice. Then the ice was polished until the surfaces were even to the edges of the ring. The galvanometer was not of sufficient sensitivity to give appreciable deflections when the nicol prism was in the system. So the nicol prism was removed and the absorption determined for the mixed ray in the region from 1.42 μ to 1.68 μ. This curve is shown in Fig. 4. The position of maximum absorption was found to be at 1.5 μ. For water measured on the same instrument the position of maximum absorption of the corresponding band was found to be at 1.44 μ.

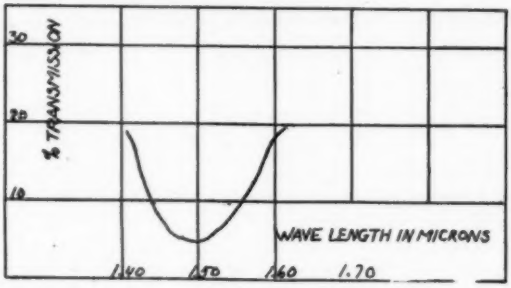

FIG. 4

One specimen of ice 5 mm thick was cut by accident so that it was made up of two crystals. The surface of separation between the crystals was perpendicular to the incident beam and the optic axes were not in the same direction in the two crystals. For this specimen of ice the absorption bands of the extraordinary ray and the ordinary ray were obtained and also a set of absorption bands whose position are almost the same as the bands in water at 0°.

The three sets of absorption bands just described were obtained with the nicol prism in the system and without it. The intensity of the bands would vary for different positions of the specimen, but the location of the center of the bands was the same in all cases. This curve is shown in Fig. 5. The wave lengths of the new set of absorption bands were observed at .77 μ, .85 μ and .99 μ. In all, this result was obtained in three different specimens of ice.

The writer has not found a satisfactory way of explaining the nature of these bands. It is not likely that it is due to a layer of water between the ice crystals. The small difference in wave length can be explained as due to the temperature and pressure, but it does not seem plausible

that a layer of water only a fraction of one mm in thickness between the ice crystals would have a per cent absorption in this region equal to that of 2 cm of water under normal conditions. It is seen that the absorption bands of the ordinary ray are separated from these inner surface absorption bands by twice the amount of the separation between the inner surface bands and extraordinary ray bands.

IV. Discussion

Previous reference has been made to the theory of Sutherland as to the molecular constitution of water and of ice. He assumes that water vapor is H_2O, that water is a mixture of $(H_2O)_2$ and $(H_2O)_3$ and has calculated that 16 per cent $(H_2O)_3$ is present in water at 100°C and

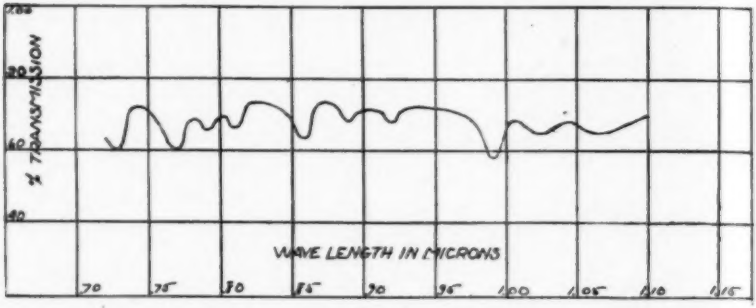

FIG. 5

37.5 per cent at 0°C. Ice is assumed to have 100 per cent $(H_2O)_3$. From Hettner's investigation we have available the position of maximum absorption in the bands of water vapor. Collins has determined the position of maximum absorption of water at 0°C and 100°C. The values for ice have been found in this work. So we can make a graph with the per cent $(H_2O)_3$ as ordinate and wave length's in microns as abscissa. In Fig. 5 is given the curves for the bands in the region of 1 μ and 1.2 μ. The values used for the wave lengths of the absorption bands of ice are for the extraordinary ray. It is interesting to note the smooth curve obtained.

G. Bode found that 1 mm of ice absorbed 93 per cent of the incident energy at 1 μ. The present work shows that 5 mm of ice at 1 μ absorbs only about 5 per cent of the energy for the extraordinary ray and about 47 per cent of the raidant energy in case of the ordinary ray. It is difficult to explain this variation in values except on the basis that surfaces of the ice used by Bode must have been very irregular and also the

ice must have contained air bubbles. There are several sources of error in the present work. First, the surfaces may not have been plane parallel. Second, the difficulty in cutting the crystal of ice so that the optic axis would be perpendicular to the beam of the radiant energy. Third, the crystal of ice might not be set so that the electric vector of the nicol prism was parallel or perpendicular to the optic axis. However, the values given in Table I and also shown in the transmission curves do not contain an error of any appreciable size. This conclusion is made upon the basis that in all the various trials there was never a variation over a few per cent.' In Fig. 3 it is observed that the general absorption is present in the ordinary ray at 0.75 μ. This would cause one to expect that the general absorption continues in the visible spectrum for the

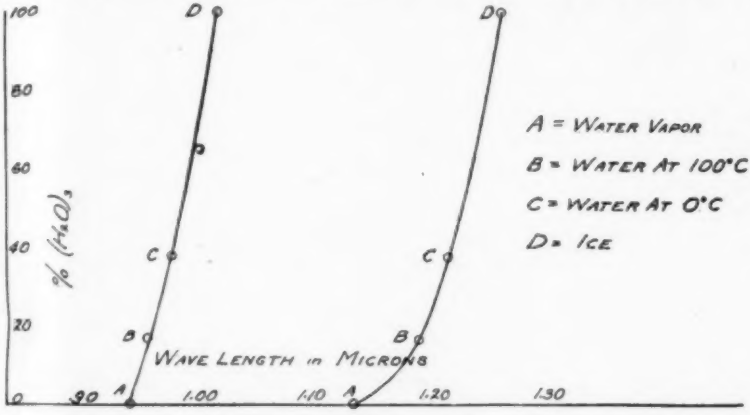

FIG. 6

ordinary ray. At 0.7 μ the transmission was 60 per cent. The deflections of the galvanometer were not sufficient to determine this region accurately so the results are not included. However, the writer expects to study the absorption of ice for the two rays in the visible spectrum and for the other bands in the near infrared. The present results are sufficient to show that the absorption of ice in the near infrared region is dependent upon the direction of the plane of polarization of the incident energy.

The writer wishes to thank Professor J. R. Collins for his encouragement and for many helpful suggestions given during the progress of this work.

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Gold from Mercury.—A large number of newspaper comments have been made recently on the supposed disintegration of mercury into gold by Miethe. The basis of these comments is a preliminary and quite indefinite note published by Miethe in Die Naturwissenschaften. He and his assistant Dr. Stammreich have examined the non volatile residue obtained after operating a Jaenicke mercury vapor arc for many hours. There is no question but that the residue in certain cases contained gold. Haber, who made the analyses, but who disclaims any participation in the work, makes the following statement. "One of the seventeen samples was free from gold and silver. A second contained a trace of gold (10^{-9}g Au per g Hg) and no silver. In all other cases there was either only some silver but no gold, or, besides small quantities of gold, a silver content of from one to two orders of magnitude greater than the gold content." Apparently considerable care was taken to see that the original mercury and the electrodes of the lamp were free from gold although no mention is made of possible contamination by the quartz walls. Miethe suggests that the supposed disintegration of Hg is due to the ejection of a helium nucleus or four hydrogen nuclei from Hg^{201} leaving Au^{197}. He does not consider that this should give an element two columns to the right of Hg requiring a subsequent emission of 3 electrons from the nucleus to produce Au, nor does he suggest the more plausible transmutation by driving a single electron into the mercury nucleus, as discussed by Soddy and by Foote. Miethe makes no comment on the presence of the Ag reported by Haber, although it would seem that it required as much explanation as the Au. He was unable to detect gold in an ordinary Hg arc and attributes his success to the fact that with the Jaenicke arc a potential of 170 volts may be applied to the electrodes.

As it would be surprising if the voltage drop per mean free path were much different in the various types of commercial arc, the electronic collisions were on the average probably in the neighborhood of 5 volts, which is the order of magnitude of ordinary chemical forces. If mercury atoms were unstable under such conditions this fact should have been detected long ago by spectroscopic and chemical analysis.

The only possible explanation is that the supposed disintegration is due to the very infrequent long range electrons which, hypothetically at least, may occasionally traverse the full length of the arc before colliding with a mercury atom.

Lacking sufficiently detailed accounts to enable any final judgment to be formed, we may, of course, speculate on the various possible sources of the Au and Ag. Certainly, however, Miethe has not presented satisfactory evidence for disintegration. Some of the discussions in the technical press are illuminating. For example, Kaul, Die Metallborse, p. 1432 gives a theory for the synthesis of atoms from hydrogen, helium and CH, CH_2, CH_4, CH_8. [Miethe, Die Naturwissenschaften, *12*, pp. 597-8, 1924; Haber, idem p. 635; Soddy, Nature *114*, p. 244, 1924; Foote, Sci. Mon. *19*, pp. 239-62, 1924.]

PAUL D. FOOTE AND L. B. TUCKERMAN

INSTRUMENT SECTION

STRING-GALVANOMETER OSCILLOGRAPH WITH AUTOMATIC PHOTOGRAPHY

By Augustus Trowbridge

Since the publication of W. Einthoven's original paper on the string galvanometer in 1903 the instrument designed by him has been used in various modified forms and for a variety of purposes: for example, as a substitute for the oscillograph in electrical measurements of alternating currents; as a detector of transient currents in land and submarine telegraphy; during the late war as a chronograph in connection with the location of artillery positions by sound waves; and as a recording instrument for use in connection with power indicators for internal combustion engines.

In the course of these adaptations of the original instrument various improvements have been made in the direction of simplicity of operation and the substitution of automatic features for those requiring individual care and attention so that at the present time it is possible to construct an instrument of this type of considerable sensitivity and yet robust enough to place in the hands of a person who exercises the ordinary care required in the use of any other form of galvanometer. It has been my experience that students, laboratory assistants or machinists learn to use a form of photographically recording string galvanometer very much more quickly than they do the more familiar form of oscillograph.

A recording string galvanometer consists of four parts: (*a*) a source of light; (*b*) a timing device; (*c*) the galvanometer proper; and (*d*) a camera for recording both the time and the motion of moving parts (generally two separate moving parts) of the galvanometer.

(*a*) For practical reasons it is advantageous to be able to use an ordinary nitrogen filled electric filament bulb as a source of light rather than the arc light which is stronger in actinic rays but which requires more attention. The use of the filament light requires the use of especially sensitive paper or film in the camera (*d*) but in my experience it is more satisfactory to use the special paper and a reliable source of light rather than an ordinary commercial paper and a special and frequently unreliable source of light.

(*b*) As a timing device a small induction motor fitted with a large number of narrow spokes and driven and governed by an electrically operated tuning fork is very reliable and cheap and simple to build. The number of poles of the motor and the number of spokes carried by the armature may be chosen with reference to the frequency of the driving and governing fork so that once every one one-hundredth of a second a spoke cuts off the light from the source and by the use of appropriate lenses a very narrow and sharp shadow is cast on the photographic film or paper of the camera. For convenience in counting every tenth spoke may be made broader or narrower than the others. The accuracy of this method of recording time intervals on a rapidly moving photographic film or sensitive paper has been investigated by Dadourian who has found that the errors from all probable causes do not exceed .002%.

(*c*) The galvanometer consists of an electromagnet giving a uniform field of as high an intensity as possible and one or two fine metallic filaments in a vertical plane at right angles to the direction of the magnetic field. The beam of light from the lamp to the camera is carried through holes in the pole pieces of the magnet and one lens is fitted in the pole piece nearest to the lamp in a position to illuminate strongly the metallic filaments, while another lens is fitted in the other pole piece in a position to focus a real image of the shadows of the metallic filaments in the plane in which the photographic paper in the camera moves. The metallic filaments which constitute the moving parts of the galvanometer should have the best conductivity and the smallest mass which can be obtained in practice (actually in commercially available materials this is at present a tungsten wire 0.0004 inch in diameter and having for the length used a resistance of about 20 ohms. It is annealed after mounting in a suitable holder by passing it through the yellow flame of a match. It is then quite free of kinks and may be adjusted in tension and position in the field of view by appropriate slow motion screws fastened to the piece which holds it and which is carried by the pole pieces of the magnet). It is important to design the electromagnet so that it will have a strong and uniform field, not only to secure a high degree of sensitivity for the instrument, but also to insure proper damping to the motions of the filament since this damping is only very slightly due to air friction and almost completely to electromagnetic causes. If a current is sent through either (or both) of the filaments it moves at right angles to its own direction and to the direction of the magnetic field in which it is situated and the

shadow of the filament thus moves at right angles to the direction of motion of the photographic paper of the camera. Just in front of the paper of the camera is placed a cylindrical lens of short focal length with its axis horizontal (i.e. at right angles to the direction of the shadow of the filament). This lens converts the long vertical shadow of the filament into a point shadow in the plane of the photographic paper so that a motion of the filament results in the motion of a point shadow on the photographic paper. If both paper and filament are moving a curve will be traced on the paper and when this is developed one has a trace of the position of the filament at any instant of time and the time itself is recorded in one hundredth second intervals by the shadows of the spokes of the wheel of the induction motor situated near the source of light. With the tungsten wire of 0.0004 inch diameter it is easily possible to obtain a motion of the shadow of the filament on the photographic paper amounting to one millimeter when a current of one one hundred thousandth of an ampere flows through the filament. However, to secure satisfactory damping to the filament so that it shall not show traces of its own free vibrations, it is usually necessary to operate it on a permanent shunt resistance of from 5 to 10 ohms so that the effective sensitivity of the instrument is 1 mm deflection for a current of from 2×10^{-5} to 4×10^{-5} amp. As the resistance of the filament is about 20 ohms this corresponds to a voltage sensitivity of one millimeter deflection on an applied voltage of from 10^{-4} to 2×10^{-4} volts. It is very easy to adjust the instrument so that it is practically dead beat and will give a deflection of a millimeter for a thousandth of a volt and about ten times this sensitivity may be attained if considerable care be taken in the adjustment.

(*d*) The most elaborate part of the apparatus is the recording camera. This should consist of a space to store a roll of sensitive photographic paper, the necessary rolls to feed it rapidly in a vertical plane at the proper distance from the cylindrical lens mentioned in (*c*) above, a space to store the exposed portion, rollers to engage the forward end of the exposed portion and begin to feed it at a proper speed through the developing tank and from this into the fixing solution and so to the hand of the man who is operating the instrument. The details of how this may be done in a satisfactory and reliable manner are given in a later illustration. Here will be given merely the results which have been obtained. It has been found possible to take a record on which the time interval of one one-hundredth of a second is drawn out on the record to the length of one inch; to develop and fix this record;

and to begin to deliver it to the operator within ten seconds after taking. At this high rate of speed the record shows quite sufficient contrast to be satisfactory for visual examination. For photographic reproduction in a magazine article, however, a taking speed of about one half this (i.e., one half inch for one one-hundredth of a second) is about the limit. Those reproduced in this article were taken at about this speed. It is impossible to obtain these results with ordinary photographic paper and with the ordinary developers. However, the Eastman Kodak Company makes up a special paper which they perfected during the war for a special purpose. This paper retains its great sensitivity for several months and is perfectly satisfactory to use and not much more expensive than the commercial papers.

Fig. 1 is a reproduction of a photograph of the complete instrument mounted on a cast iron base about a meter in length and about 20 cm wide. At the left of the figure is shown the automatic developing camera which consists of an enamelled tank with two compartments.

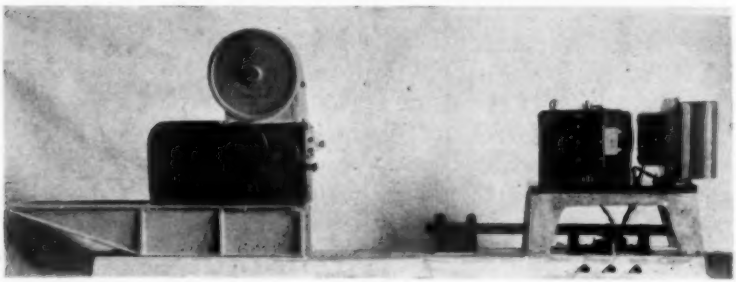


FIG. 1

The compartment directly under the main part of the camera contains the developer and the triangular compartment at the left contains the fixing solution. Since the exposed film or paper is carried through the developing tank with its sensitive side down and is fed into the fixing solution in this position it is sufficiently protected from the further action of light and may be fixed in the diffuse light of the room. The circular container shown at the top of the camera unit holds the unexposed supply of photographic paper. It is sufficiently large to hold rolls of about 200 feet in length and is designed to take in various widths of paper up to about 2½ inches. The portion of the camera unit which appears dark in the reproduction is a heavy brass casting which contains the various rollers and gears necessary to feed the paper from

the magazine at any desired speed up to about 100 inches per second and to feed it at any desired fraction of this speed through the developing solution. This portion of the camera also contains a knife which cuts the photographic paper at the conclusion of an exposure. In addition there is ample storage space for the exposed paper which accumulates in the cases in which the speed of taking a record is great and the speed with which it is being developed is slow.

Fig. 2 shows the front end view of the camera with a rather narrow strip of paper coming from the magazine above and engaged by one of the rollers. When exposures are being taken the rollers are kept turning by an electric motor but the paper is not fed forward until a lever is thrown by the operator. When this lever is thrown the roller behind which the paper is seen to disappear in Fig. 2 is thrown backwards and the paper is caught between it and a similar moving roller directly behind it. The paper is then fed rapidly forward and an exposure is made. The front end of the paper is caught between other rollers (not visible in Fig. 2) and commences to be fed at slow speed into the developing tank while the surplus exposed paper is guided into the storage space. When the exposure is finished the lever is thrown back and this time it not only disengages the driving rollers but also cuts the paper at a point about an inch beyond where the paper disappears from view in Fig. 2. Thus the feeding of unexposed paper ceases when the lever is thrown back but the exposed paper beyond the cut continues to be fed through the developer until the entire record which may be a meter or more in length has been passed through the camera.

Fig. 2 shows the door, which is normally closed when an exposure is being made, which carries a wide slit faced with velvet against which the moving paper passes under slight pressure applied from behind. This insures the constancy of the plane of motion of the paper. The cylindrical lens mentioned in (*c*) and (*d*) above is carried on the door and in addition there is an adjustable slit to control the amount of light which falls on the cylindrical lens. Only two sets of rollers are used in the camera, one set to control the speed with which the exposure is made and the other to control the speed with which the paper is developed. The paper is pushed (not pulled) through the developer. Pieces of paper over a meter in length may be pushed when wet with the greatest ease. There is not the slightest tendency to fold and of course no possibility of tearing, whereas, at least as far as my own experience goes, attempts to pull wet paper through the developing and fixing tanks resulted in a large percentage of ruined films.

Mounted on the main base of the instrument is an electrically driven tuning fork with two sets of contacts, one set for maintaining the vibration ofthe fork and the other for supplying electrical current impulses to the spoked wheel mentioned in (*b*) above. The fork with its contacts is shown toward the right of Fig. 1. Above it on a casting in the form of a table is mounted on the right a unit consisting of the

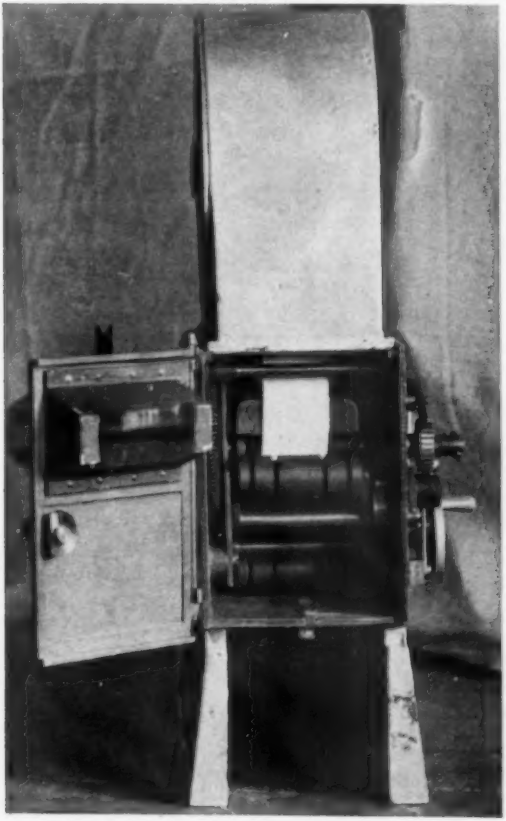


FIG. 2

source of light protected by a cylinder and the timing device mentioned in (*b*) above. Between this and the camera is the galvanometer unit. Above and below may be seen the adjusting screws for one of the filament holders; also, to right and left may be seen the lens holders in the holes through the pole pieces.

The binding posts for the source of electrical supply are shown on the right of the main casting on which the instrument is assembled. This electrical supply is a 24-volt storage battery (2 twelve-volt or 4 six-volt automobile batteries in series). The tuning fork and spoked wheel are kept running continuously when the instrument is in use; also the current which supplies the field of the galvanometer magnet. But in order to not unduly shorten the life of the electric light which must be used at a considerable over voltage to secure adequate light for high speed films, it is well to control this and the motor which runs the camera by a foot switch on which the operator steps at the moment an exposure is being made.

The electrical connections are so simple as not to need a diagram to describe them. Current is drawn from a storage battery of 24 volts (4 six-volt automobile batteries in series). The coils of the electromagnet are designed so that there is practically a saturated field when the battery is applied directly to the coils and the field current may be left closed indefinitely without overheating. The same battery operates the electrically driven tuning fork in the usual manner by means of a coil and vibrating contact. A second vibrating contact also attached to the fork supplies properly timed electrical impulses from the common battery to the synchronous motor which carries the spokes which make the shadow lines of the time scale. The 12-volt nitrogen filled straight filament electric bulb which serves as a source of light is also supplied with current from the 24-volt battery and by means of the foot switch mentioned above which enables one to apply an excess voltage (about 21 to 22 volts) to the 12-volt lamp for the very short time that is required to make an exposure.

The motor which drives the camera may either be supplied from the common battery, or, if 110-volt dc or ac power be available, by a separate source of power. The operation of the camera may be advantageously controlled by the same foot switch which controls the source of light.

The optical system is given diagrammatically in the accompanying sketch in which L is the source of light; l_1 a short focus lens which gives a real image of the light source at i_1; l_2 an inexpensive microscope objective mounted in one of the pole pieces of the galvanometer magnet and which serves to illuminate brightly the strings s_1 and s_2 of the galvanometer; l_3 is a high grade microscope objective of about 30 mm focal length mounted on the other pole piece and which serves to bring sharp aplanatic and achromatic images of the shadows of the strings

s_1 and s_2 to a real focus in the plane of the photographic paper P. At about 1 inch distance in front of the photographic paper is placed the cylindrical lens C with its axis horizontal. This serves to bring the line image of the shadows of the strings s_1 and s_2 to point foci of these shadows in the plane of the photographic paper P. At S between the lamp L and the short focus lens l_1 is the horizontal position of any one of the spokes of the timing wheel run by the synchronous motor. By means of the various lenses a sharp line image of the shadow of any horizontal spoke is brought to a sharp focus on the photographic paper P at the instant when the spoke is in the horizontal position. In this manner the succession of instants when the spokes of the time wheel are in a horizontal position are recorded on the moving paper P. Also the instantaneous position of each of the strings s_1 and s_2 are recorded

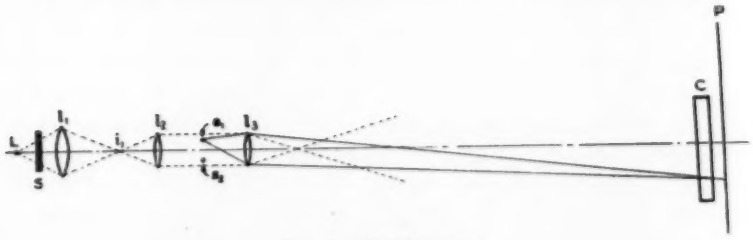


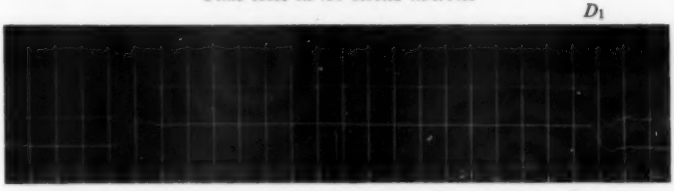
FIG. 3. *Optical system.*

as points of shadow in the paper P and these points, in virtue of the motion of the paper are recorded as lines giving the position of the strings at every instant of time. It should be noted that since the optical magnification of the lens system is considerable the motion of the shadow image of the string on the paper P corresponds to but a very small actual motion of the string itself so that for motions of several centimeters of the image of the string the actual motion of the string itself is only a small fraction of a millimeter for which the change of tension of the string is but very slightly altered.

In the following illustrations a few sample records are shown. Naturally there has been a loss of distinctness in the process of reproduction. The original records are all 2 inches wide and about 3 ft. long (the portion reproduced being about 8 inches in length). In every case even in record D_3 where the motion of the string is very rapid, it is perfectly easy to see the trace on the original records throughout its entire length with the unaided eye.

Record D_1 shows a "make" followed by a "break" of a current of about 10^{-4} amperes through two strings in series. The interval between the long transverse lines is .01 second and early times are on the left of the figure and late times are on the right. The string shown at the bottom of the figure is about properly damped while that shown above has not been so well adjusted for damping.

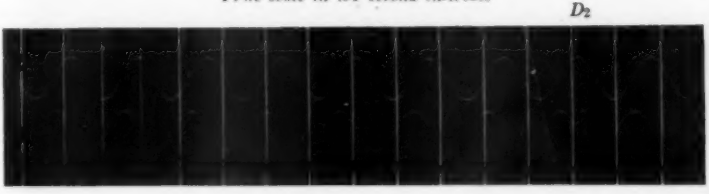
FIG. 4. *Records D*
Time scale in .10 second intervals



Make and break of a current of about 10^{-4} amperes in two strings in series. Damping in upper string not satisfactory; damping in lower string better

Record D_2 in which the photographic paper was moving about twice as rapidly as in record D_1 represents, above, the alternating potential difference across a paper telephone condenser through which the alternating current, shown below, was flowing. The frequency of the alternating current was 60 cycles. The lead of current with respect to voltage is evidently very closely a quarter period.

Time scale in .01 second intervals

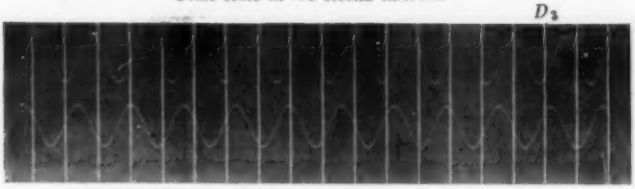


Voltage (above) over and current (below) through a condenser. Alternating emf, 60 cycles

Records D_4, D_5, and D_6 are indicator diagrams of the pressure variations in the cylinder of four cycle internal combustion engine. The revolutions per minute of the engine were 720 in D_4, 1086 in D_5, and 1248 in D_6. The engine was kept on about one half throttle and was loaded heavily. The spark setting was very much retarded in D_4; it was set slightly too retarded for best running in D_5 and slightly too advanced for good running in D_6.

In these records the position of the string does *not* represent the pressure in the cylinder but rather the rate of change of the pressure with the time $\frac{dp}{dt}$. Thus where the excursion of the string is greatest the pressure is rising most rapidly; when the excursion is greatest downwards the pressure is falling most rapidly: where the string is in its undeflected position the pressure is constant.

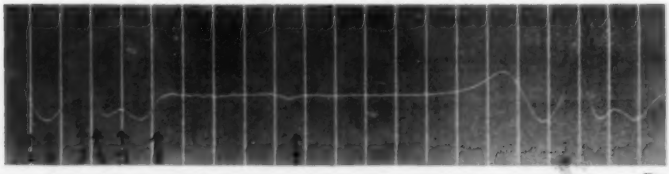
Records D (continued 2)
Time scale in .01 second intervals



Current (below) through and voltage (above) over an inductive resistance. Alternating emf, 60 cycles

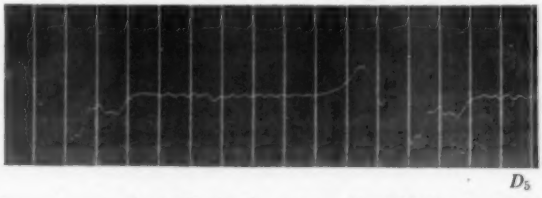
In record D_4 the times at which various events in the cycle take place are shown by numbered arrows in the margin, thus: 1, explosion in-center; 2, spark; 3, maximum rate of increase of pressure; 4, maximum pressure; 5, exhaust valve begins to open; 6, out-center; 7, intake valve opening.

Records D (continued 3)



About ½ throttle engine loaded spark very retard; rpm 720.

In record D_5 the spark occurs 13° of rotation of the crank-shaft *after* in-center and the maximum pressure comes 60° after in-center.

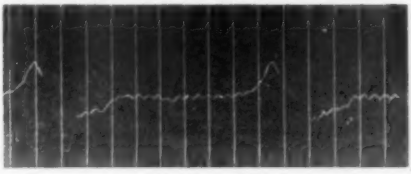


Maximum pressure about 60° after in center. About ½ throttle-engine loaded spark about 13° after in center; rpm 1086.

In record D_6 the spark occurs 10° *before* in-center and the maximum pressure 25° after in-center.

The slow quiet burning of the mixture in D_4 and the turbulent explosive ignition in D_6 is evident from the character of the two records.

The values given above are only approximate since in these records the actual phase of the motion of the crank-shaft is not recorded. This may be done accurately by using the second string in connection with a small circuit breaker mounted on the crank-shaft so that the in-center position of any given cylinder may be marked on the record. In this way it is possible to time very accurately any event in the cycle even when the engine is running with wide open throttle at between two and three thousand rpm.



D_6

Maximum pressure about 25° in center. Maximum pressure about 25° after in center. About ½ throttle-engine loaded spark about 10° before in center rpm 1248.

For the study of the running condition of a high speed internal combustion engine the form of indicator diagram shown in records D has marked advantage over the ordinary indicator diagram giving the pressure as a function of the volume of the gas in the cylinder. Such an indicator diagram crowds the events which take place near in or out center (spark, opening of valves) whereas a diagram which gives the pressure, or which gives the rate of variation of the pressure with the time, as a function of the time, has a much more open scale so that events taking place near the in or out center positions may be timed much more accurately than they can be on the standard p,v diagram.

For the accurate timing of an event the $\frac{dp}{dt}$, t diagram has a farther advantage with respect to the p,v or with the p,t diagram in that a change of pressure is merely shown in the p,t diagram as a change in slope, whereas in the $\frac{dp}{dt}$, t diagram it is shown as a "nick" in the curve and may thus be the more readily noticed and also the more readily timed.

In the study of the running engine one is generally more interested in the changes produced by changing conditions than one is in the absolute values of the pressure in the cylinder at every instant, so the fact that records of the character *D* do not give directly the value of the pressure is not a practical disadvantage to this type of indicator diagram. It has a very marked advantage in the study of *changes* in running conditions due to changes in valve or spark setting or to changes in mixture, temperature, etc.

It is not contended that a string galvanometer has any theoretical advantages over the more common form of oscillograph. For certain types of work a moving member whose free frequency is vastly greater than that of any frequency which it is proposed to measure is decidedly preferable to one which is adjusted to be aperiodic. What is contended is, however, that with extremely simple means one can construct an oscillograph employing the string galvanometer principle which in a great variety of cases is quite satisfactory, which possesses the advantage that the time scale is quite independent of the uniform running of the motor which drives the film, and which has the further advantage that the record is delivered in permanent form almost instantly, thus saving the operator's time and preventing any possibility of the confusion of records which is always a possibility when a number of photographs are taken and subsequently developed.

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A STUDY OF PFUND'S PRESSURE GAGE

By R. T. Cox

ABSTRACT

Pfund's combination of the McLeod and Pirani pressure gages was studied with a number of gases. The calibration curves of pressure against galvanometer reading were found to be straight lines and, except for hydrogen, nearly interchangeable for all the gases used. By using a galvanometer of high sensitivity and introducing a compensating filament, the sensitivity of the gage was increased until a change in pressure of 1.5×10^{-9} mm of mercury would cause a change in deflection of 1 cm on the galvanometer scale.

Brief comments are made on the release of carbon monoxide and air from coconut charcoal and on the adsorption of air at low pressure by silica gel.

INTRODUCTION

A. H. Pfund[1] has described a combination of the McLeod and Pirani[2,3] gages in which the capillary tube of a McLeod gage is made the gas chamber of a Pirani gage in order to detect pressures existing there too small to cause any appreciable difference in the heights of the mercury columns. By this means a pressure sensitivity is obtained whose order is that of the product of the sensitivities of the two gages used separately.

The theory of the McLeod gage is too simple and too well known to bear repetition here. A brief description of the Pirani gage may not be out of place.

At ordinary pressures the conduction of heat through a gas is effected by successive impacts of molecules with one another, for at these pressures the mean free path of the molecules is small in comparison with the dimensions of the containing vessel. When this is the case, it may be shown from kinetic considerations that the thermal conductivity of a given gas is independent of the pressure. An increase in pressure produces, it is true, an increase in the number of molecules available for transferring heat, but this effect is exactly counteracted by the shortening of the mean free path. But if the pressure be so low that the mean free path is comparable with the dimensions of the vessel, this counteraction no longer takes place and the thermal conductivity increases with the pressure. Hence at such a pressure an electrically heated filament will be more rapidly cooled the greater is the pressure of the gas surrounding it. If a constant current be main-

[1] A. H. Pfund. Phys. Rev. Series 2, *18*, p. 78; 1921.
[2] M. v. Pirani. Ber. d. Deutsch. Phys. Gesell. *4*, p. 687; 1906.
[3] C. F. Hale. Trans. American Electrochem. Soc. *A 20*, p. 243; 1911.

tained in such a filament the temperature and hence the resistance of the filament will diminish as the pressure is increased. The resistance of the filament may thus be used to indicate the pressure.

In Pfund's gage, the filament is sealed in the tip of the capillary of a McLeod gage, so that its resistance indicates the pressure obtained by compressing the gas from the bulb up into the capillary. The sensitivity of the McLeod gage is thus increased as many fold as the Pirani gage is more sensitive than a simple mercury manometer.

At the suggestion of Professor Pfund, the experiments herein described were undertaken with the following purposes:

1. To compare the calibrations of the gage for different gases.
2. To find how the gage should be constructed and how used to give the most convenient operation and the highest precision.
3. To reach, if possible, the sensitivity of 3.4×10^{-10} mm of mercury named by Pfund as a probable limit for this gage.

The difficulties met in carrying out the first of these purposes were expected to serve as guides in effecting the second and third, which are obviously closely allied. Hence the procedure outlined in describing the first experiments is not that which the author has learned to prefer.

Apparatus

The apparatus used differed from that used by Pfund chiefly in the parts accessory to the gage itself. It is shown in Fig. 1. The gas to be studied was admitted to the apparatus through a tube containing phosphorus pentoxide to absorb water vapor. Plugs of glass wool in the ends of this tube prevented the scattering of the phosphorus pentoxide through the apparatus and aided a gradual diffusion rather than a sudden rush of the gas, thus giving the phosphorus pentoxide a greater time in which to act. By means of a three-way stopcock the rest of the apparatus could be opened to this tube and the gas supply or could be sealed off from them. The same stopcock made connection with the pump. This was a rotary oil pump capable of reducing the pressure to 10^{-3} mm of mercury. The trap containing coconut charcoal immersed in liquid air was used to purify helium by absorbing any other gases present. The gas discharge tube was introduced to make possible spectroscopic tests of the purity of the gas admitted. By means of the mercury seal, the gage could be cut off from the part of the apparatus on the left, which, it was thought, might contain organic vapors from the stopcock grease or other impurities. The purpose of the gas chamber

was to render negligible in comparison with the total volume the changes in volume caused by raising the mercury in the gage. Its lower end served as a vessel for coconut charcoal immersed in liquid air, which was used in obtaining the highest vacua measured and served also to remove impurities. The fall tubes of the mercury seal and the gage ended in traps designed to keep the mercury clean. Impurities gathered on the free surface of the mercury in the trap instead of rising to the top of the mercury column. Any gases, also, percolating through or given off by the rubber tube holding the mercury, gathered in the recess above

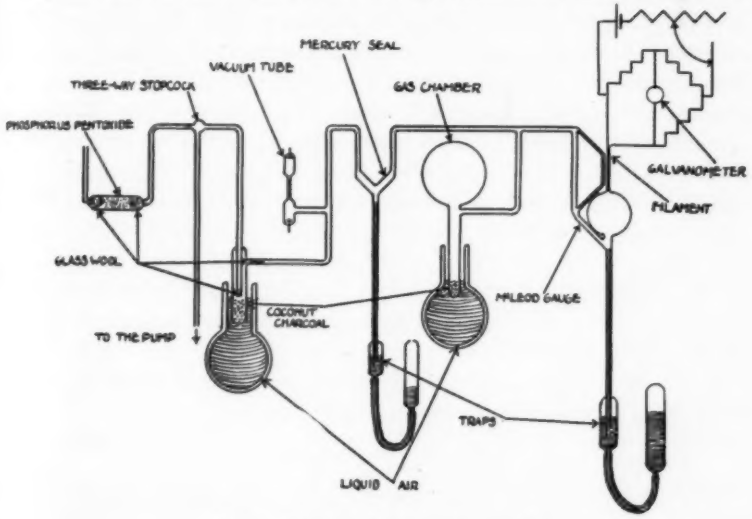


FIG. 1. *Diagram of gage and accessories.*

this surface rather than enter the evacuated parts of the apparatus. To further protect the mercury from contamination, the rubber tube was lined with short sections of glass tubing. That these might fit tightly they were fed in from a long metal cartridge while the rubber tube was inflated with compressed air. The bulb of the McLeod gage had a capacity of 182 cm.^{3} The capillary, 10 cm in length, had a capacity of .0086 cm^3 for each centimeter of length. Its internal diameter was thus about 1 mm. A hairpin filament of tungsten wire about two mils in diameter extended 3 mm down the capillary. Any contact, except at the seal, between this filament and the wall of the capillary would have imperilled the constancy of the calibration and probably impaired the sensitivity of the gage. To prevent such contact Professor Pfund de-

vised the following means. A wire, tapered and bent to form a hook at the upper end and weighted at the lower end, was thrust up the capillary and engaged in the filament. The capillary was then evacuated and the filament heated electrically until it became limp and was drawn straight by the weight. It was then allowed to cool and remained straight. The filament formed one arm of a Wheatstone bridge having three fixed arms of "Therlo" wire approximately equal in resistance. The bridge was balanced by varying the current through it, thus changing the resistance of the filament. Variable resistance for the fine adjustment of the current was furnished by a heavy brass wire with a sliding contact. All the junctions of the bridge except at the binding posts of instruments had to be soldered. During the operation of the gage, the current through the bridge was about 300 milliamperes. When the current was increased to 400 milliamperes the filament glowed red in a darkened room. The filament was protected against external thermal effects by putting two glass plates in front of and behind the capillary and enclosing these in a celluloid cylinder closed at top and bottom with cotton batting.

The resistance of the filament was so sensitive to minute changes in the current that the form of bridge just described was discarded in later experiments.

Before being used, the apparatus, from the stopcock to the gage, was heated with a flame to drive off occluded gases.

Method of Calibration

The gage was calibrated by finding the pressures in the capillary corresponding to different deflections of the galvanometer in the bridge. It was assumed, and the assumption tested, that the galvanometer deflection depended only on the nature and pressure of the gas in the capillary and not on the height at which the mercury stood. The bridge was balanced at the lowest pressure obtained. Then, the mercury in the gage standing below the bulb so that the bulb was open to the rest of the apparatus, gas was admitted until a convenient deflection was read on the galvanometer. The pressure in the bulb, the same of course under these conditions as that in the capillary, was then measured by the usual method with the McLeod gage. Then more gas was admitted and the deflection of the galvanometer again noted. The pressure corresponding to this increased deflection was then measured. This was continued until the deflection corresponding to the greatest pressure measurable with the McLeod gage was found. The calibration curve

was drawn showing pressure in the capillary plotted as ordinate against galvanometer deflection as abscissa. This curve could then be used to measure very low pressures by compressing a highly rarefied gas into the capillary until a convenient galvanometer deflection was obtained. The pressure corresponding to this deflection was read from the curve. The product of this pressure by the ratio of the volume of the portion of the capillary occupied by the compressed gas to the volume of the bulb was, by Boyle's law, the pressure initially prevailing in the bulb, just as in the usual use of the McLeod gage. This, of course, involves the assumption that Boyle's law holds at pressures less than any at which it has been tested. Should future experiment, however, indicate a departure from Boyle's law at these pressures, the usefulness of the gage will not thereby be destroyed. It will be necessary only to regard the gage as measuring density rather than pressure. For, however the pressure of a given mass of gas may be found to vary with its volume, it is evident that the density must vary inversely with the volume. Conversely, a gage of this type can not be used to determine the validity of Boyle's law at low pressures.

Gas	Length of Capillary Used	Pressure
	mm	mm
Hydrogen	50	2.2×10^{-6}
	36	2.4
	27	2.3
	20	2.3
	12	2.3
Air	98	7.2
	80	7.1
	65	7.1
	46	7.2
	38	7.6
	27	7.0
	19	7.3
	18	7.2
Oxygen	75	2.1
	49	2.1
	31	2.4
	18	2.3
	12	2.3
	5	2.3
Carbon Dioxide	35	4.8
	13	4.8

The assumption that the deflection depends only on the nature and pressure of the gas present and not on the portion of the capillary which the gas occupies was tested as follows: Let it be supposed that the assumption is incorrect. Then it is altogether reasonable to believe that it will be more incorrect the smaller the volume occupied by the gas, i. e., the nearer the mercury approaches the tip of the filament. Hence pressure determinations made with the mercury high in the capillary will not agree with those made with the mercury far down in the capillary. The table gives determinations of the same pressure as measured with the mercury at different levels from near the bottom of the capillary to within a few millimeters of the tip of the filament.

Calibration with Different Gases

In the manner just described (slightly modified in certain cases) the gage was calibrated with helium, hydrogen, air, oxygen, carbon dioxide, and carbon monoxide. These gases were obtained and prepared as follows:

The helium used was part of a supply furnished this laboratory by the Navy Department. It contained a certain amount of nitrogen, which was removed by coconut charcoal in liquid air. The hydrogen and oxygen were obtained by the electrolysis of water and freed from water vapor by passage through phosphorus pentoxide.

The air was not treated intentionally except with phosphorus pentoxide. It is likely, however, that the carbon dioxide content was depleted by the coconut charcoal in liquid air used to produce the highest vacua.

Carbon dioxide was produced by the action of hydrochloric acid on marble. To free the carbon dioxide from acid it was collected in a vessel inverted over a solution of sodium carbonate. It then entered the apparatus through the phosphorus pentoxide tube.

Carbon monoxide was obtained by heating in vacuo a mixture of potassium ferro-cyanide and sulphuric acid, allowing it to cool, and then admitting the liberated gas through the phosphorus pentoxide tube to the apparatus.

The calibration curves for hydrogen and carbon dioxide are shown in Figs. 2 and 3. These and the other curves obtained are straight lines to within the error of measurement. It was also observed that the slopes of the different curves are not greatly different; indeed with the exception of the curve for hydrogen they are very nearly equal. (The slopes are the only significant feature of the calibration curves. The

intercepts on the horizontal axis are determined by the galvanometer zero, normally at 0.5 on the scale, and the pressure at which the bridge was balanced, normally the lowest attainable.) The agreement between the slopes is clearly shown in Fig. 4, in which two points from each calibration curve except that for hydrogen are plotted on the same sheet. It is possible to draw a straight line which passes as near to these points as most of the calibration curves pass to the points by

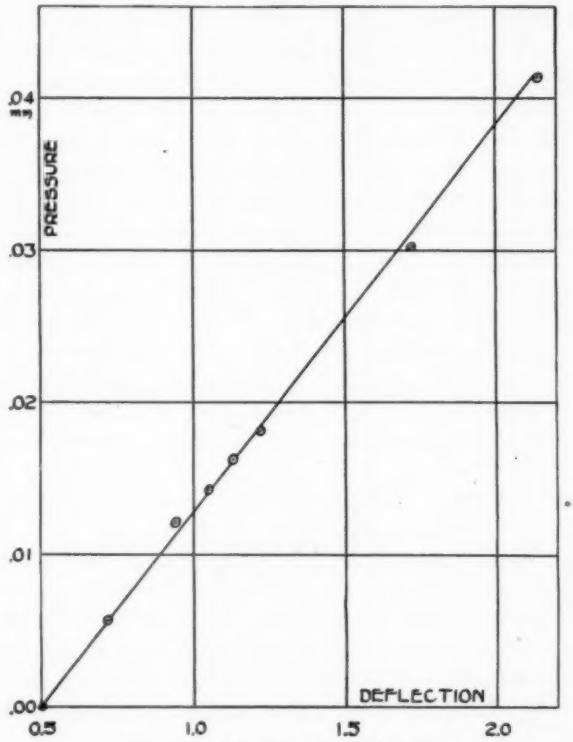


FIG. 2. *Calibration curve with hydrogen.*

which they are plotted. For most work a single calibration curve would serve for all the gases used except hydrogen. Indeed in much of the work at these extreme low pressures it is only desired to know the order of magnitude of the pressure. In such work even the difference between the curve obtained for hydrogen and the curve shown in Fig. 4 would not be significant.

The relative slopes of the curves obtained for air and hydrogen agree with the results obtained with a hot-wire gage by C. F. Hale, in so far as wide differences in the construction and use of the apparatus permit comparison between his results and those here described.

The sensitivity of the gage, expressed in scale divisions per millimeter of pressure, is simply the reciprocal of the slope of the calibration curve. Since the deflection results from change in the filament resistance

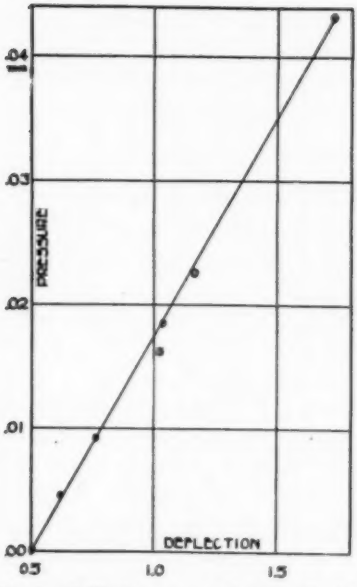


FIG. 3. *Calibration curve with carbon dioxide.*

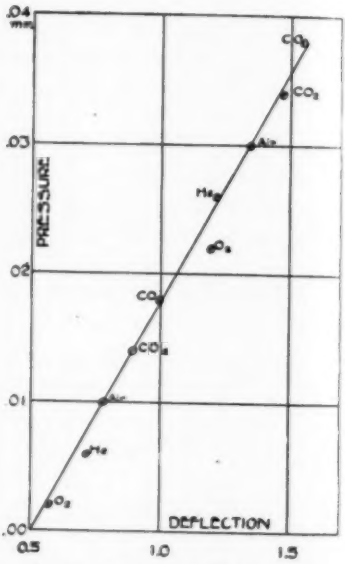


FIG. 4. *Composite calibration curve.*

which, in turn, is caused by the conduction of heat from the filament, it follows that the more highly conducting a gas is, the greater will be the sensitivity obtained with that gas. The following table shows the sensitivities obtained with the gases used.

Gas	*Sensitivity*
Helium	25
Hydrogen	39
Air	28
Oxygen	31
Carbon Dioxide	31
Carbon Monoxide	27

It is of interest to compare these data with the observations of F. Soddy and A. J. Berry[4] on the conduction of heat through rarefied gases. Their conclusion that the heat conducted through a rarefied gas varies directly with the pressure is reflected in the linearity of the calibration curves as shown in Figs. 2, 3, and 4. The small difference found in the present investigation between the sensitivities with different gases is in accord with their observation "that the conductivity found for gases at low pressure varies far less with the nature of the gas. . . .than is to be expected from kinetic considerations." (*Conductivity* is here used in a special sense, viz., the number of calories conducted from one square centimeter of heated surface per second per .01 mm pressure per degree Centigrade temperature difference between the heated surface and its surroundings.) Their observation that the conductivities at low pressures bear no relation to those at high pressures is supported by the sensitivities tabulated above. As regards the relative conductivities of different gases, the fact observed by Soddy and Berry that the conductivity varies with the temperature and the likelihood that it is affected also by other factors such as the shape and size of the bounding solid surfaces and the presence or absence of mercury vapor probably render agreement and disagreement alike insignificant. In fact, comparing the differences in sensitivity found with different gases with the errors of measurement, the author of the present paper is not himself disposed to attach much importance to these differences other than that between the sensitivity with hydrogen and the sensitivities with other gases. Soddy and Berry find helium, oxygen, carbon dioxide, and carbon monoxide about equally conducting, a result in accord with the values of sensitivity given above. On the other hand, their observation that nitrogen is almost as conducting as hydrogen and much more conducting than the other gases named is in apparent conflict with the sensitivities found with air, hydrogen, and the other gases, as is also their observation that helium is more conducting than oxygen, carbon dioxide, or carbon monoxide.

RELEASE OF CARBON MONOXIDE AND AIR FROM COCONUT CHARCOAL

When the coconut charcoal was allowed to adsorb a fair quantity of carbon monoxide and subsequently made to expel it by the application of heat the behaviour of the charcoal was highly interesting. About five grams of the charcoal in pieces about the size of BB shot rested

[4] F. Soddy and A. J. Berry. Proc. Roy. Soc. *A 83*, p. 254; 1909.
ibid. *A 84*, p. 576; 1910.

in the bottom of a tube about 2 cm in diameter. When a flame was applied to the bottom of the tube for a short time, the greater part of the charcoal rose suddenly like a rigid body to a height of perhaps 10 cm, where it lodged until shaken loose by tapping the side of the tube. It then separated into a mass of violently agitated particles, some of which were thrown up to a height of about 25 cms and hovered there for some time. Although the charcoal had originally been sifted, quantities of charcoal dust were produced, probably by friction between the large pieces. This action went on almost continuously during about half an hour of heating and seemed by no means ready to subside at the end of that time. The pump was running continuously during these observations. A pressure of about 0.1 mm was maintained. The same observations were repeated later, this time with the apparatus sealed. The original quasi-rigid motion of the charcoal again occurred, followed by the same violent agitation. It was noticed that to produce these effects the charcoal had to be heated from beneath; if the tube were uniformly heated around the sides little motion took place and motion started by heating at the bottom actually subsided even during a very brisk heating of the sides. If one region only on the side of the tube were heated, the particles were driven away from that region. These observations suggested that the motion of the particles was like that of a rocket, that it was a recoil produced by the molecules leaving the heated side of the particle. An instantaneous view of the whole mass seemed to show the particles grouped in striations. After five or ten minutes of heating, the pressure had increased to 1 mm. It was not observed that the motion increased with the pressure as might be expected if the phenomenon were one of convection.

The apparatus was again pumped out and sealed and heat was again applied. The motion of the particles was resumed. A clearly defined cloud of dust rose and fell in the tube with a period of about one second. At the end of twenty minutes the agitation seemed somewhat lessened. The pressure had increased from .004 mm to 1 mm, which indicated that .003 g of gas had been given off. During this time an estimated average mass of 3 g of charcoal had been kept in suspension in the tube. The impulse required for this was 35×10^5 dyne-seconds. If, on the hypothesis that the charcoal is supported by the recoil of the departing gas molecules, this impulse be ascribed to the momentum of these molecules, we arrive at the altogether incredible conclusion that the speed of their departure is greater than 10^9 cm per second.

Exactly the same behaviour was later observed when adsorbed air was driven off from the charcoal. To see if the phenomenon were electrical, pieces of foil were wrapped around the tube and charged by means of an electrostatic induction machine. No change in the behavior of the particles was observed.

This phenomenon seems to merit further observation. The fact recorded by a number of observers that oxygen adsorbed by charcoal is re-evolved, on heating, partly as carbon monoxide and carbon dioxide may be pertinent.

CHANGE OF SENSITIVITY WITH FILAMENT TEMPERATURE

It was thought that a marked increase of sensitivity might result from an increase in the filament temperature. The filament was accordingly heated to a dull glow and observations taken as before. The gas used was air. It was found that the linearity of the calibration curve persisted even at this temperature. But the sensitivity when the current through the bridge was great enough to produce dull incandescence was not found to be much higher than when the current was reduced to about three-fourths of this value.

MODIFICATIONS TO INCREASE SENSITIVITY

As has been stated, the chief source of error in using the bridge was the disproportionate change in the deflection caused by small changes in the current through the bridge. To do away with this difficulty recourse was had to the device suggested by Pirani and used by Hale wherein the bridge resistance on the same side of the bridge as the filament in the gage is furnished by an exactly similar filament in a tube sealed at a very high vacuum. This compensating filament thus undergoes the same changes in resistance caused by changes in current as does the gage filament. In the present case it was mounted alongside the gage filament and was thus also subjected to the same external thermal conditions. Hence deflections are to be ascribed to changes in the only factor not shared by the two filaments, viz., the pressure of the gas in the capillary. (The deflection is, of course, proportional to the current through the bridge and changes in this current will produce proportionate changes in deflection, but these changes are negligible in comparison with the disproportionate changes which the compensating filament is designed to eliminate.) The chances of obtaining precise compensation are impaired by the difficulty of duplicating exactly a filament 6 mm long and 2 mils in diameter, sealed in a capillary

1 mm in diameter. The compensation obtained was by no means perfect, but the steadiness of the deflections was nevertheless markedly increased. With the compensating filament in the bridge it was, of course, no longer possible to balance the bridge by varying the current through it. One of the other arms was therefore made variable. A low-resistance wall galvanometer of moderate sensibility was now substituted for the portable galvanometer. It was found that a pressure in the capillary of .0047 mm produced a deflection of 63 cm on the galvanometer scale. Taking one centimeter as the least significant reading, we have for the least pressure measurable by the hot-wire gage alone 7.5×10^{-5} mm. The ratio of the least volume of the capillary which could be used to that of the bulb was 2×10^{-5}. Hence the least pressure measurable by the gage as a whole was $(2\times10^{-5})\times(7.5\times10^{-5})$ or 1.5×10^{-9} mm. The author was not able to produce a pressure as low as this.

The gage so refined was found to be very sensitive to thermal changes. To prevent such changes all the larger vessels were wrapped in cotton batting.

Comparison of Pfund's Gage with Other Gages

It is of interest to compare this gage with some of the other gages used for the measurement of extremely low pressures. A gage designed and used by Langmuir[5] and later studied by Dushman[6] makes use of the change in the viscosity of a gas which takes place as the pressure reaches very low values. The viscosity, like the conductivity, of a gas is independent of the pressure when the mean free path is small in comparison with the dimensions of the containing vessel. But at pressures at which the mean free path is comparable with these dimensions, the viscosity varies with the pressure as does the conductivity. The Langmuir gage is rather intricate in structure. It consists essentially of two disks in a glass bulb. The upper one is suspended by a fibre from the top of the bulb. It carries a small mirror. The lower disk is mounted on frictionless bearings and carries a magnet by means of which it can be rotated at an adjustable speed by a rotating magnetic field produced outside the bulb. Because of the viscosity of the gas the lower disk, when rotated, exerts a certain drag on the upper one. A beam of light reflected from the mirror shows the deflection of the upper disk. This deflection is a measure of the viscosity of the gas and

[5] I. Langmuir. Phys. Rev. Series 2, *1*, p. 337; 1913.
[6] S. Dushman. Phys. Rev. Series 2, *5*, p. 213, 1915.

hence of the pressure. For moderately low pressures the calibration can be obtained by extrapolation from a comparison at higher pressures with a McLeod gage. For extremely low pressures the calibration is calculated from kinetic theory, a procedure open to question if the viscosity departs from the calculated values to the same extent as the conductivity. Dushman's observations indicate that this gage can be used to indicate pressures as low as 10^{-7} mm, the limit originally predicted by Langmuir.

A number of gages have been built on the principle of Crookes' "radiometer". The most sensitive of these seems to be Shrader and Sherwood's[7] modification of Knudsen's[8] gage. In this, as in the other forms of Knudsen's gage, a vane suspended in a glass bulb is subjected to bombardment by the molecules leaving an electrically heated strip whose distance from the vane is less than the mean free path of the gas molecules. Under these conditions the deflection of the vane caused by this bombardment is related to the pressure of the gas in the bulb by an equation derived by Knudsen from kinetic theory. The constants of this equation depend only on the mechanical construction and operating conditions of the gage and can, moreover, be measured directly. The calibration is thus absolute and the same for all gases. With a gage of this type pressures as low as 2×10^{-8} mm of mercury were observed and it was estimated that pressures as low as 5×10^{-9} mm could be indicated.

Buckley's[9] ionization gage has the advantage over those just described, especially Langmuir's, of being simpler in construction. The lowest pressures thus far observed with it are, however, of a higher order than those observed by Dushman with Langmuir's gage and by Shrader and Sherwood.

A striking difference between Pfund's gage and the others mentioned is that Pfund's gage does not indicate the pressure of the mercury vapor or other condensible vapors present, since the effect of these pressures is automatically eliminated in the calibration. This is an advantage or a disadvantage according to the nature of the investigation in which the gage is used. Probably it is more often desirable than undesirable to eliminate this effect. Obviously the gage can not be used in any investigation in which the presence of mercury vapor would defeat the purpose of the investigation.

[7] J. E. Shrader and R. G. Sherwood. Phys. Rev. Series 2, *12*, p. 70; 1918.
[8] M. Knudsen. Ann. d. Phys. *32*, p. 809; 1910.
[9] O. E. Buckley. Proc. Nat. Acad. Sci. *2*, p. 683; 1916.

The range of pressures measurable with Pfund's gage is most remarkable. By using an extra long capillary or, better yet, by blowing a small bulb in the capillary near its lower end to increase its capacity, the range of the modified McLeod gage can be made to overlap the range of the unaided McLeod gage. The range of the unaided gage can be made in turn to overlap the range of pressures which can be measured by comparing the height of the mercury column in the long vertical tube of the gage with the height of the barometric column. A single instrument might thus measure pressures from one atmosphere to 2×10^{-12} atmosphere.

Adsorption at Low Pressure by Silica Gel

Through the courtesy of Dr. Eberman of the Johns Hopkins Chemical Laboratory, a sample of specially activated silica gel was obtained and tested for its ability to produce high vacua. It was put in a tube which was sealed in the apparatus in such a way that it was open to the gage. The gas used was dry air. After the silica gel had been immersed in liquid air for an hour and forty minutes, it had reduced the pressure only from .0034 mm to .00064 mm, and of this adsorption perhaps a third is to be ascribed to adsorption on the glass walls of the container. In less time than this, an equal volume of properly prepared charcoal would produce a vacuum a hundred times as high. It is of course possible that another sample of silica gel would show altogether different properties.

Acknowledgment

The author is under great obligation to Professor Pfund, who not only suggested the investigation just described but furnished indispensible help and advice throughout its course. To Professor Ames, the Director of the Johns Hopkins Physical Laboratory, and to several members of the department he is indebted for their aid and interest. He is especially grateful to Mr. Joseph Kaplan, who aided in much of the construction of the apparatus and in taking observations and who was most unsparing of his time and labor.

The Johns Hopkins University,
Baltimore, Maryland,
June 24, 1924.

ACOUSTIC WAVE FILTERS IN SERIES

By G. W. Stewart

It is interesting to inquire if certain limitations of a single acoustic wave filter cannot be eliminated by the use of filters in series. As may be anticipated from the approximate nature of the theory of action of acoustic filters and as shown by experiment[1] there are additional transmission bands in both the low frequency and single band pass filters. These bands are caused[2] by the resonance of each section of the transmitting conduit.[3] Hence it is possible to place in series two filters with unlike additional bands and hence with attenuation in that region. This is a report of the experimental test of such an arrangement.

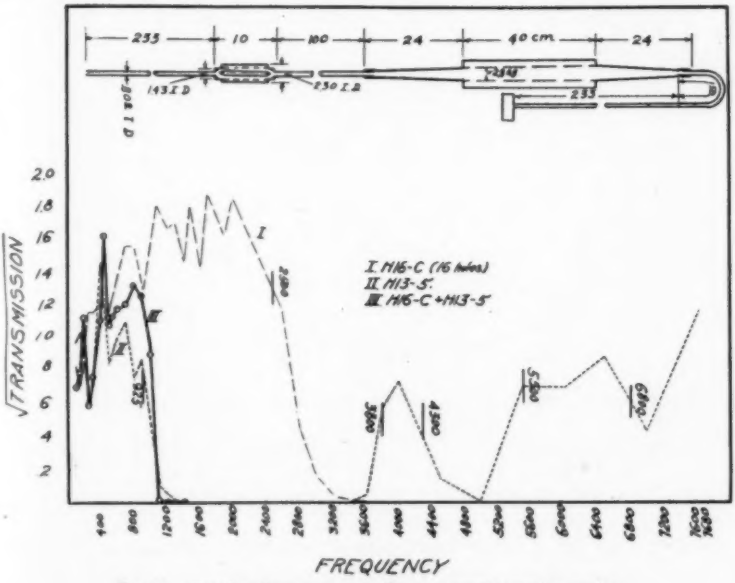


Fig. 1. *Two low frequency pass filters separately and in series.*

Fig. 1 shows the individual transmission curves of two low frequency pass filters. Curve I shows a cut-off frequency at 2500 and (theoretically) an additional band at 8720 to 10300, outside of our range. Curve

[1] Stewart, Phys. Rev. 20, No. 6, Dec. 1922.

[2] Theory to be published.

[3] Resonance in the branch line above the fundamental will also produce a band, but not in the cases herein or earlier discussed by the author.

II shows a cut-off frequency of 925 and additional bands at 3800 to 4300 and 5500 to 6800, these being the theoretical values. There is theoretically a transmission band at 1700 to 2150 but it is too narrow to be in evidence experimentally. The bands for the two filters do not coincide and the result of placing both filters in series is Curve III. The transmission greater than unity is caused by reflections as is also the oscillating nature of the curves. These reflections are introduced by changes in dimensions of the conduit as indicated in the drawing in Fig. 1. Careful arrangement of equipment would produce much more satisfactory curves.

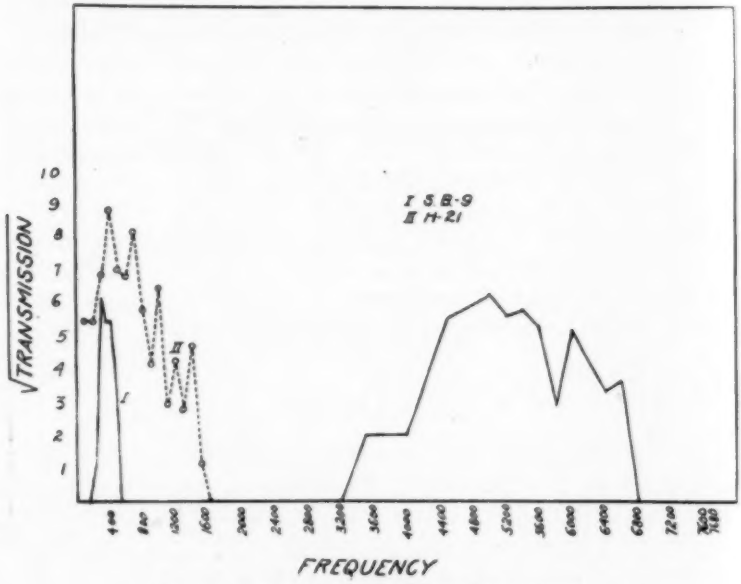


FIG. 2. *A single band and a low pass filter separately.*

Fig. 3, Curves I and II, indicates the combination producing a single band filter with remarkably good attenuation over a range ten times its upper limit. Curves I and II in Fig. 2 show the respective transmission curves of a single band and a low frequency pass filter which when placed in series give Curves I and II of Fig. 3.

These acoustic wave filters terminate in conduits and because the impedance there cannot be the same as the characteristic impedance of an infinite filter, reflection must occur diminishing the filter action.

That this effect is not distinctly noticeable is shown by the sharpness of the cut-off of the combination. Also when the damping tube 100 cm in length was removed from between the filters, shown in Fig. 3, the resulting transmission Curve II of Fig. 3 was almost identical with Curve I.

Another interesting point is that these filters in series give greater transmission in the "non-attenuated" region than would be expected from the individual transmission curves. It is surmised that while this

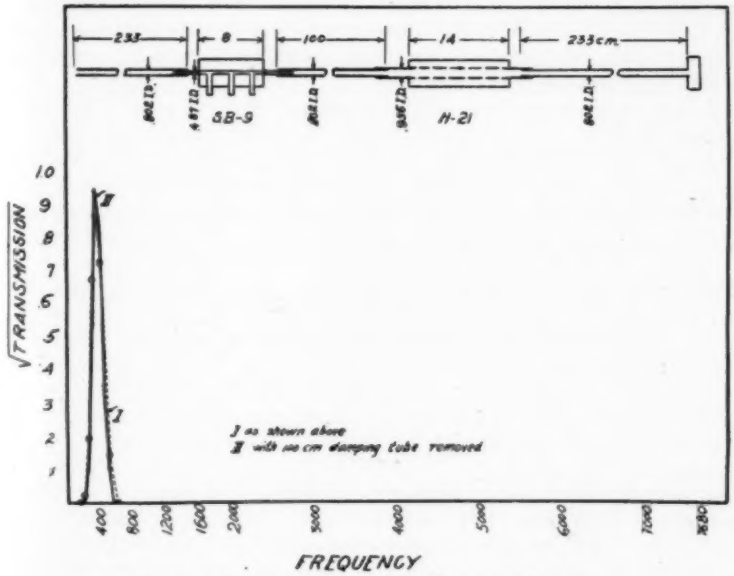


FIG. 3. *The two filters and their series transmission curve.*

may be caused by interaction of the filters, yet a more probable cause is the variation in reflections aside from the filtering action.

These experiments indicate indefinite possibilities in the combination of acoustic wave filters.

I wish to acknowledge with appreciation the assistance of Mr. R. V. Guthrie in these experiments.

UNIVERSITY OF IOWA.
IOWA CITY, IOWA.

Light and Work. By M. Luckiesh. Published by D. Van Nostrand Co. 289 pp. Price $4.00. 70 illustrations.

The book is a delightfully written treatise on light and work, not so much a partial development in physics as it is a mixture of general science and philosophy showing the influence of light of varying character upon the development of man and the relations between illumination and human efficiency.

The author adopts a biological viewpoint in which he sets forth that man's visual equipment as well as his habits of work and recreation were developed under natural illumination and he postulates therefore that artificial light that simulates the best choice of natural illumination is safe. In the author's words, "We can not without risk subject our sense organs and mental being to conditions radically different from those for which they have become tuned."

The book is written in fifteen chapters but the subject matter is about equally divided in three parts. About one third treats of the nature of natural and artificial light, another third treats physiological and psychological aspects of vision while the remainder builds on these considerations and is a very convincing discussion of quality and quantity of light in relation to effective vision and efficient work.

There are explanations of a mass of facts such as why the sky is not black, why artists think they prefer light from a north sky, why folks generally are attracted to a fireplace for recreation and rest at evening time, why diffusing glass should not be used in windows, why green illumination is soothing etc.

The work is more than a composite of physics, engineering, biology and philosophy. Its greatest contribution is in economics. After showing that artificial illumination is almost invariably faint compared with natural light, the author proceeds to show in an interesting manner that the installation of better lighting for workers is generally a paying investment. He gives data showing that the value of increased production in factories far exceeds the cost involved. In the chapter on the economic intensity of illumination there is much information that should be useful to illumination engineers such for example as the expenditure that would be warranted for improving a lighting system for wage earners receiving from thirty to ninety cents per hour.

The book should be useful to engineers and architects and everyone interested in correct illumination. On the other hand those cultivated in science generally will want to read it. It is a good reference book for any physics library and I would like to suggest its consideration for a textbook in general science.

F. C. BROWN

A PHOTOGRAPHIC RECORDER FOR TEMPERATURE AND OTHER PHYSICAL QUANTITIES

By W. H. Stannard and Paul E. Klopsteg

Several times within the past few years development problems have confronted this laboratory in which the data on a number of variables had to be obtained simultaneously over long periods of time. With only one or two observers available, incomplete data only could be obtained, and while the observers were engaged in securing them the theoretical work, computation, and other aspects of the problem were out of the question. The instrument to be described was developed for the purpose of obtaining the data mentioned much more completely and accurately than is possible by personal observation, and at the same time to relieve the research engineer of the tedium of making the observations, thereby releasing him for the other important work involved in the problem.

An application of the recorder has been made to the study of electrically heated devices in which the temperature is to be maintained constant by automatic means. Although the description which follows refers specifically in some of its illustrations to the study of temperature control, the apparatus can obviously be used in the study of any problem in which it is possible to obtain galvanometer deflections as known functions of the variables in question. The recorder as finally worked out has fulfilled every expectation and has proved so convenient that a description may be helpful to others who encounter problems similar to ours.

General Scheme

The plan adopted employs thermoelements in conjunction with two reflecting galvanometers and a roll of photographic paper. This involved nothing entirely new except the details of handling the paper, and it was, therefore, thought to be quite practicable. It has been developed with no more difficulty than was expected. As the complete recording apparatus is complex, its description is for convenience treated under the following headings: Optical Systems, Mechanism and Operation, Photographic Details, Electrical Circuits.

OPTICAL SYSTEMS

The optical system for each of the two galvanometers is so arranged that an intense beam of light is projected from a concentrated filament lamp by way of the galvanometer mirror to the photographic paper. Above the paper is a cylindrical lens which concentrates the beam of light to a small spot. As the galvanometer coil is deflected, the spot moves along the focal line of the lens across the paper. To differentiate the records from the two galvanometers the widths of the corresponding spots are made different.

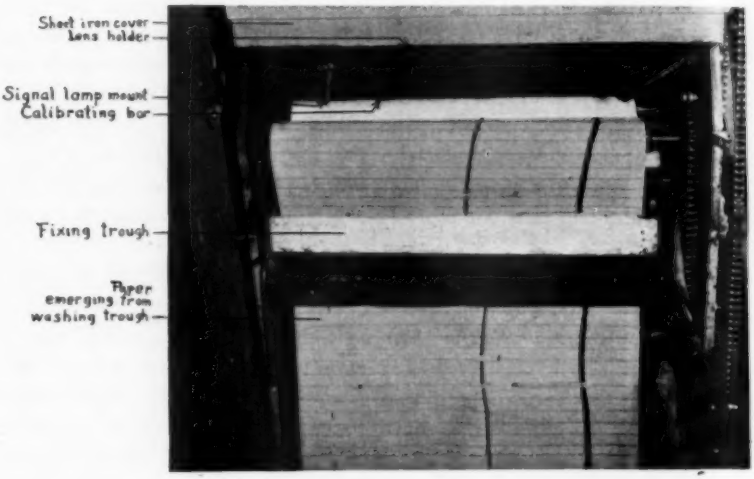


FIG. 1 *View of recording parts of the mechanism. The white horizontal strip near the top of the picture shows the photographic paper which has been exposed but not developed. Just below this strip is shown a section of paper which has been developed and rinsed, and which is passing into the fixing bath. In the lower section of the picture is shown the paper after it has been fixed and washed. The paper is dried between the time it emerges from the washing trough until it arrives at the roll on which it is wound.*

Time lines are produced on the paper by means of a signal lamp, which shines once a minute by reason of a connection to an electrical clock system. The lamp is mounted between the galvanometers, and the image of its filament, produced by the cylindrical lens, is a well defined line across the paper. The other scale of coordinates is produced by breaks in this line, caused by the shadows of the teeth of the

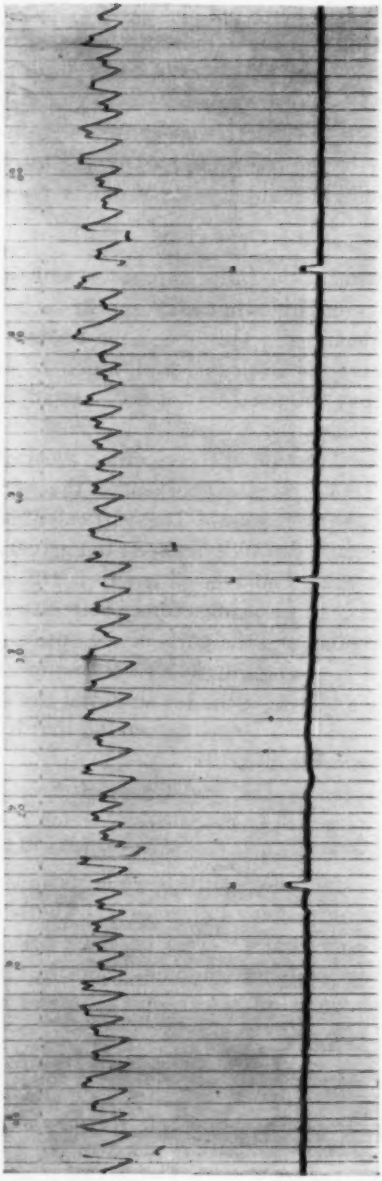


FIG. 2. *Typical record from automatic recorder. Direction of travel of the paper is toward the left. Figures at top of chart indicate time. Spaces between parallel lines represent minutes. In the present instance vertical spaces represent one degree each or a total of ten degrees between bottom and top of the chart. The short dashes in the horizontal line near the top of the chart indicate the time intervals during which a heating unit was thrown into the circuit. It will be noted that each one of these corresponds to an abrupt rise in the upper graph. This graph represents the temperature of the sensitive member of a thermoregulator controlling oven temperature while the lower graph represents the temperature of a small quantity of material in a drying dish in the oven.*

The abruptly displaced portions occurring simultaneously in both graphs represent the working zero positions of the two galvanometers. The vertical distance between the zero for the upper graph and the displaced portion occurring two minutes later gives an indication of the percentage error in the potentiometer working current.

"calibrating bar." The calibrating bar is parallel to the cylindrical lens, and its front edge is just behind the line of light on the paper from the timing lamp. It is virtually a comb with eleven teeth projecting forward and spaced 25 mm apart across the whole width of the paper. The general result of the whole optical arrangement is seen in Fig. 1, which shows a close-up view of the recording part of the mechanism, and Fig. 2, which is a facsimile in reduced scale of one of the records.

MECHANISM AND OPERATION

To insure stability of adjustment the entire apparatus, and especially its optical system, was constructed in what may appear to be an unnecessarily heavy and rigid manner. This was done because of the shocks to which the laboratory is subjected and which might with lighter construction quickly put the parts out of accurate adjustment. The results obtained have justified the design.

Fig. 3 shows a median vertical section of the recorder, illustrating its mechanical arrangement. To economize floor space the horizontal dimensions were reduced to the minimum. The vertical dimensions were made large in order to secure distance between the galvanometer mirrors and the recording surface. The over-all length is 185 cm. Its foundation is a 2″ by 6″ steel channel beam securely bolted to a heavy brick wall; on it the galvanometers and the parts of the optical system are rigidly mounted as shown in the figure. The recording mechanism is assembled as a unit which is bolted to the lower end of the channel beam. The cylindrical lens is held in a hinged mount, so that it can be swung vertically upward when occasion arises.

The galvanometers, prisms, lenses, and upper part of the recording mechanism are protected by a light-tight metal cover, which is secured to the channel beam in such a manner that it is easily removable to give access to the galvanometers. Hand holes in this cover permit adjustment of the galvanometer zeros without its removal. The holes are closed during operation of the recorder. Two one-inch brass tubes attached to the sheet metal cover permit the light from the lamps to pass to the galvanometers, but cut off extraneous light and exclude dirt.

Fig. 3 also gives a clear picture of the operation of the recording mechanism. A roll of photographic paper 10 inches wide and 100 feet long, sensitive side in, is placed upon the rollers A, B in the magazine. The paper travels as shown in the diagram. It is developed at F and rinsed at H. The 75-watt drying lamp dries the paper before it reaches

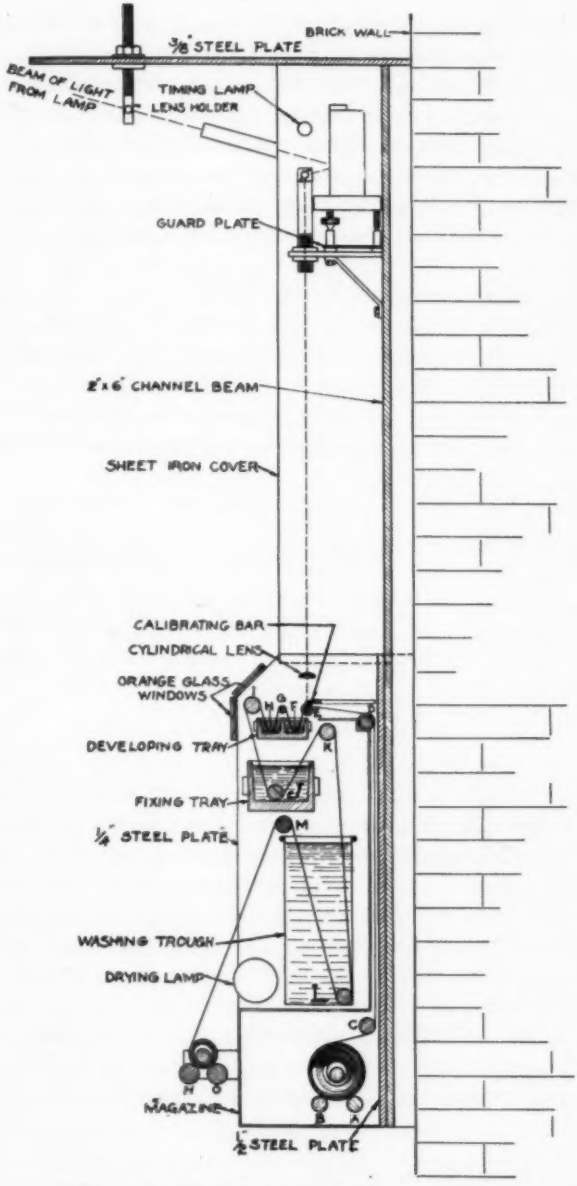


FIG. 3. *Median vertical section of recorder mechanism.*

the receiving rollers *N*, *O*. Rollers *F*, *H*, *J*, and *L* merely hold the paper down in their respective compartments by their own weight. Rollers *A*, *D*, *E*, *G*, *I*, *K*, *M*, and *N* are driven in synchronism by means of sprockets and chain. Rollers *B* and *C* run idle. The paper is fed at the rate of about one cm per minute. It was foreseen that it would be impossible to turn all the driven rollers so accurately to size that the paper would pass continuously over them without slip and without producing undue slack or tension. It was not foreseen that the wet paper would adhere to the rollers so firmly that it would tear rather than slip. This failure to slip necessitated special means to effect automatic compensation for inaccuracies in the rollers. The plan adopted was, first, to provide a positive drive for the paper through only one roller, second, to drive the other driven rollers at excessive peripheral speeds, and, third, to provide means that would cause the paper automatically to lag behind the surfaces of the rollers by the proper amounts. This plan proved altogether successful.

At some convenient time during each run the operator marks on the paper at the calibrating bar the hour and minute of the day together with a line representing the beginning of that minute (as indicated by a watch). This is done, without admitting light to the paper, by the use of a piece of graphite in a small chuck on the end of a steel rod. The rod is so bent that it reaches from the operator's hand up behind the windows and back over the developing tray.

Photographic Details

The paper used is a "gaslight" paper, 10 inches wide, in rolls of 100 feet. The developer, which is held in a five-gallon bottle on a high shelf near the recorder, is conducted through tubing to a point just above the developing compartment, where it drips into one end of this compartment. There is an overflow at the opposite end of the compartment, so that the developer enters at one end and passes over the entire width of the paper and out at the other end. The rate of dripping is regulated by means of a glass stopcock to about three drops per second.

The rinsing compartment of the developing tray also has an overflow at one end, and the water is introduced in a small stream at the opposite end.

The paper is fixed, washed, and dried as indicated in Fig. 3.

Electrical Circuits

The principal electrical circuits used with the recorder are: (1) the measuring circuits proper including the potentiometer wire and poten-

tiometer battery and rheostat; (2) the checking circuits, which are temporary modifications of the measuring circuits used for checking the zeros of the galvanometers and the standard cell balance; (3) the leakage-guard circuit; and (4) the signal lamp circuit. The accessory circuits, including those of the motor, galvanometer lamps, and relay magnets need no description. The others are described in the following sections, and are with the exception of the signal lamp and guard circuits shown in detail in Fig. 5.

MEASURING CIRCUITS

The recorder is being used for measuring small temperature changes at temperatures ranging from room to several hundred degrees Centigrade. It is, therefore, necessary to employ a potentiometer arrangement of circuits. The arrangement is shown in simplified form in Fig. 4. In a potentiometer a circuit carrying a constant, known current is

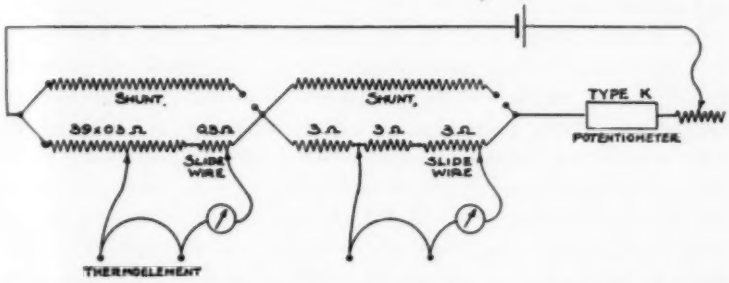


FIG. 4. *Schematic diagram of circuits which constitute the potentiometers for the thermoelements.*

needed, and, as a Leeds & Northrup type K potentiometer is part of the laboratory equipment, it is used for maintaining and measuring the current in the special potentiometer. For this purpose the special potentiometer is connected in series with the type K potentiometer and its battery. The special potentiometer consists of two slide-wires and a specially built resistance box, which is divided into two parts (Fig. 5). In use the type K potentiometer is balanced against its standard cell; this insures that a current of 0.02000 ampere is flowing through the two parts of the special resistance box and the two slide-wires. Each half of the special resistance box is divided into a number of equal units, each of which is of the same resistance as one meter of the corresponding slide-wire. Each slide-wire is made 101 cm long to provide a small overlap. In one half of the special potentiometer each unit is 0.3 ohm, and

in the other each unit is 3 ohms. Each half is also provided with a shunt, which can be closed across it by means of a plug, and which reduces the current in the working wire by a known factor. The thermoelements used so far are composed of copper and constantan. They are either single or multiple as occasion requires.

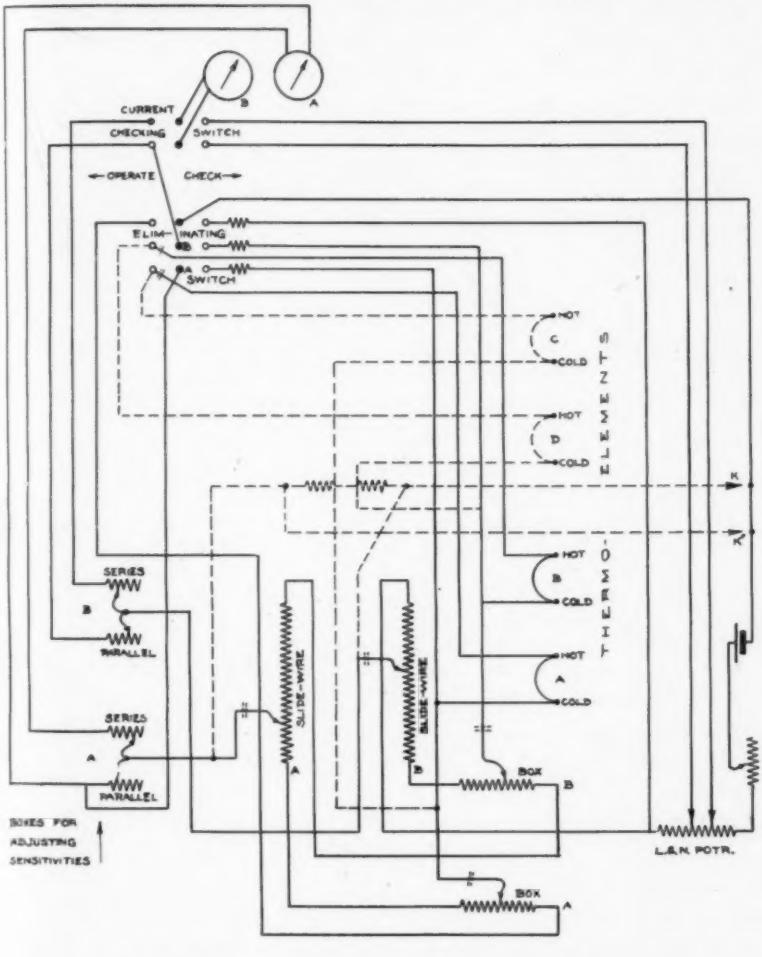
Temperature measurements made with the recorder divide themselves into two classes; first, measurements of actual temperatures, in which case the cold junctions are kept in melting ice; and, second, measurements of temperature differences, in which case both hot and cold junctions are in the apparatus under test. In the first case, the large emf's developed by the thermoelements are applied in opposition to the emf's in the potentiometer. In the second case, the generally small emf's developed are impressed directly on the galvanometers (with their series and parallel resistors), and the special potentiometer is not used. These two conditions are represented in Fig. 5 by the solid and dotted lines respectively.

In order that the scale of temperature on the chart shall be the desired number of degrees per 25 mm, the galvanometers are calibrated by adjustment of their series and parallel resistors. The constants of the thermoelements are not included in the calibration but are assumed, because great precision of calibration is not required. In Case 1 of the next preceding paragraph, the sensitiveness is measured by noting the change of galvanometer deflection, as the sliding contact is moved along the slide-wire by the number of millimeters that calculation shows is equivalent to a convenient number of degrees. In Case 2, the slide-wires being out of circuit, recourse is had to the two four-terminal resistors placed in the galvanometer circuits and shown in Fig. 5. These remain in circuit with the galvanometers; and the change of deflection, when the 0.02000 ampere is put through the resistors, is noted. The values of the resistors being known, simple calculation shows how many degrees correspond to the observed deflection.

AUTOMATIC CHECKING

As was to be expected, the zeros of the two galvanometers, especially that of the more sensitive one, wander about. This zero drift necessitates means for automatically checking the galvanometer zeros periodically. Furthermore, the potentiometer current cannot be depended upon to remain constant to the percentage accuracy to which the temperature record will sometimes be read.

It was thought impracticable to provide any automatic arrangement for holding the potentiometer current constant. Therefore, conditions



SOLID LINES REPRESENT CONNECTIONS WHEN THE THERMOELEMENTS A AND B ARE USED FOR MEASURING ACTUAL TEMPERATURES. WHEN THE THERMOELEMENTS C AND D ARE USED FOR MEASURING SMALL TEMPERATURE DIFFERENCES, THE CONNECTIONS REPRESENTED BY DOTTED LINES ARE ADDED, AND THE FOREGOING CONNECTIONS ARE SEVERED AT THE SIX POINTS INDICATED BY THE DOUBLE DOTTED LINES. LEADS K, K' ARE PLACED IN SERIES WITH THE POTENTIOMETER WHILE THE GALVANOMETERS ARE BEING CALIBRATED; AT OTHER TIMES THEY REMAIN ON OPEN-CIRCUIT.

FIG. 5

that might cause changes in the current are avoided so far as practicable, and automatic means are provided for periodically switching the

less sensitive galvanometer out of the thermoelement circuit and into circuit with the type K potentiometer and its standard cell. Thus the galvanometer deflection from zero indicates the percentage by which the potentiometer current is above or below the normal value. This switch is the double-pole, double-throw switch shown in the diagram, from which its method of operation is apparent.

The galvanometers must have their zeros checked on closed circuit, so as to take into account any parasitic emf's in their circuits. For this purpose an eliminating switch adapted from that of W. P. White[1] is used. Any parasitics existing in either galvanometer circuit remain unchanged when the eliminating switch is thrown, and hence their net effect on the record is zero.

The two automatic switches are operated electromagnetically through a contact maker geared to the driving mechanism of the recorder. The gears are so arranged that the galvanometer zeros are checked approximately once every twenty minutes, and the potentiometer current is checked about two minutes after each checking of the zeros. This automatic checking provides one row of short, straight lines for each galvanometer, which represents its working zero, and another row in the record of the less sensitive galvanometer only, which represents the error in the potentiometer current.

LEAKAGE GUARD

Because of the high sensitiveness of the galvanometers, it is necessary to take great precautions against leakage of current. The entire wiring shown in Fig. 5 is protected by a leakage guard, which consists of all the conduits, plates, etc., supporting the various parts of the potentiometer circuit. These are all electrically connected. The wiring is carefully insulated from the guard (more than 50 megohms), and the guard is also insulated from earth. In use the metal parts supporting the thermoelements are also connected into the guard circuit. The effect of the guard circuit is to isolate the entire wiring from all possible outside sources of current by an equipotential surface; hence the only possible leaks are those caused by the small voltages in the measuring circuits themselves.

RECORDING SIGNAL LAMP

Although the galvanometers are necessary for recording the quantitative variations with time of any quantity, a mere succession of events

[1] Jour. Am. Chem. Soc. *36*, 1859, 1914.

capable of causing a small lamp to flash on and off may be recorded by the action on the photographic paper of the light from that lamp. One or more such signal lamps would be useful for several purposes; such as the manual recording, through a key or switch, of an event observed or an action performed and also to record the automatic operation of any circuit closing device.

One such signal lamp is now mounted in the recorder. It is used to record the making and breaking of a heating current by a thermo-regulator. The dotted line at the top of Fig. 2 is the record of this lamp. It indicates the manner of operation of the contacts; average power consumption; and, in conjunction with the temperature record, thermal lag. Such a record is necessary for a critical study of the performance of the regulator. The lamp is a small automobile lamp partly enclosed in an inverted cone of polished sheet aluminum with a hole about one mm in diameter at its apex. The whole mount is free to rise and fall with the apex of the cone riding on the paper.

To avoid making the record more difficult to read the signal lamp record must coincide with one row of breaks in the time lines. The cone is, therefore, made to take the place of one tooth of the calibrating bar. It is tilted backward sufficiently so that only its apex obstructs the light from the timing lamp or the galvanometers, and it is placed so that its spot of light falls directly on the image of the timing lamp when the latter flashes. Thus the cone registers a break in each timing line only slightly wider than those made by the fixed teeth of the calibrating bar, and at the same time it draws a dotted line through these breaks, representing the opening and closing of the regulator contacts.

SUMMARY

Daily use of the recorder since its completion has fully justified the decision to build it for use in many of the development problems of this laboratory. Automatically and with little attention to operation it gives results which are more complete and reliable than could otherwise be obtained even by the continuous employment of two observers and a computer. In the study of electrically heated, constant temperature devices it gives accurate information about such quantities as thermal lag which could otherwise be obtained only with great difficulty. The fact that the recording elements are sensitive galvanometers makes the device very flexible, so that it can be applied to other kinds of investigation than thermal, such as that of variations in pressure, insulating

properties of dielectrics, or of any other quantity that can produce galvanometer deflections which have a known relation to the variation in question.

DEVELOPMENT LABORATORY,
CENTRAL SCIENTIFIC COMPANY,
CHICAGO, ILLINOIS.
JULY, 1924.

Electro- and Magneto-Optical Double Refraction of Suspensions.—It is found experimentally that for crystalline particles of about the same index of refraction as the liquid, (a) the electric double refraction is positive if the major axes of the ellipsoids of dielectric constant and of refractive index coincide, negative if they are normal, (b) the magnetic double refraction is positive if the major axes of the ellipsoids of permeability and of refractive index coincide, negative if they are normal. (Double refraction is considered "positive" if similar to that produced by a positive uniaxial crystal). For suspensions of fine metallic particles both electric and magnetic double refraction are negative. Both effects in all cases reach a saturation point at moderate fields.

The theory proposed is that the ellipsoids of dielectric constant and permeability determine the tendency for orientation of each particle. The optical properties of the crystal then merely superpose upon those of the liquid, and are intermediate between those of the solid crystal and of the pure liquid. Saturation results when all of the particles are aligned.

To explain the effect for the metallic suspensions it is shown that a dielectric composed of parallel rods of isotropic material in an isotropic liquid of different dielectric constant will have a different dielectric constant in different directions, thus, following Maxwell's relation with the index of refraction, should be doubly refracting. (This has been checked with diffraction gratings. However, the author admits trouble with the algebraic sign of the effect.)

There are then two effects of double refraction, one due to the internal *structure* of the particles, and the other due to the external *form* presented by the particles in the liquid.

Considering the molecules of a pure liquid as analogous to the particles in the suspension, there may then be two effects in pure liquids, which would explain the reversal of sign of the double refraction in these at a certain field strength, which has been observed before. [S. Procopiu, Annales de Physique, X, *1*, p. 213; 1924.] P. MERTZ

THERMOSTATS FOR VERY HIGH TEMPERATURES*

By Leason H. Adams

For the automatic regulation of high temperatures many devices have been used and many more have been suggested. One of the earliest successful regulators for electric furnaces made use of a thermocouple connected to a galvanometer the boom of which actuated a mechanically-operated relay, which in turn altered the current through the furnace in such a way as to keep the temperature constant. Among the promising methods suggested is that of using a porcelain bulb containing a carbonate (e.g. $CaCO_3$), or some other material with a suitable vapor pressure. The change in pressure within the bulb due to variations of temperature opens or closes a mercury contact similar to that used for low-temperature thermostats. In this brief article there is no opportunity for discussing the relative merits of various devices; it will be sufficient to describe what is without question the best method for automatic regulation of electric furnaces at high temperatures.

The principle of this method is illustrated in Fig. 1. The heating coil, or element, forms one arm of a wheatstone bridge.[1] A change in temperature of the furnace is accompanied by an unbalancing of the bridge, which applies an emf to the galvanometer and thus operates a mechanism which increases or decreases the current through the furnace. By this means a very considerable amount of energy is available for operating the galvanometer; with a platinum resistance-furnace connected to a 110 volt D.C. line, the change in galvanometer-emf for a temperature variation of 1° may be as much as several millivolts. Normally the main-line contact opens and closes once or twice a second, and the thermal lag between the heating coil and the interior of the furnace is such that fluctuations of furnace-temperature are negligible.

Fig. 2 is a more complete diagram of connections for an electric-furnace operated from a 110-volt D.C. power supply and taking not more than 15 amperes. The rheostat R_A adjusts the current through

* One of a series of reports prepared at the request of the Committee on Research Methods and Technique appointed by the Division of Physical Sciences of the National Research Council. It is the purpose of these reports to present in brief outline a summary, more or less critical, of the methods which may be employed in various kinds of Physical measurements. See Editorial in this Journal *9*, p. 410, 1924.

[1] White and Adams, Phys. Rev. *14*, pp. 44-48; 1919.

the furnace F. The resistance of R_7 may be 1 ohm and of R_5 75 ohms. The slide wire D_1, for fine adjustment of balance-point of the bridge (and hence of the furnace-temperature) has a resistance of, say, 0.11 ohm. D_2, for coarse adjustment, is a switch-arm with 29 buttons and 28 coils of such resistances that the effect of one step is always $\frac{1}{11}$ of the effect of the whole range of D_1. With R_6 equal to 3.5 ohms the first coil of D_2 (the one nearest R_6) is 0.35 ohm; each succeeding coil is 10 per cent greater than the one it follows, the 28th coil having a resistance of 4.55 ohms and the total resistance of D_2 being 46.5 ohms. With these values the maximum and minimum resistances of F are 38 and 1.4 ohms respectively.

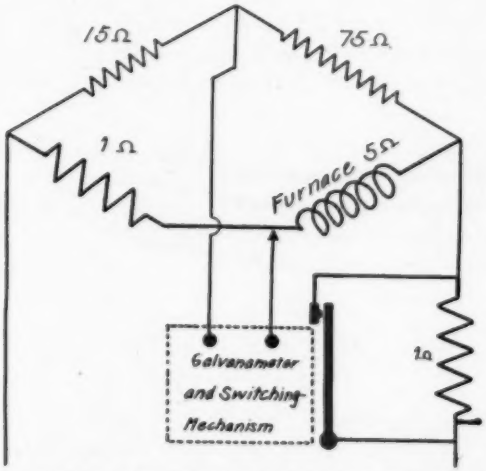


FIG. 1. *Diagram to illustrate principle of temperature-regulator for electric-furnaces. The heating coil forms one arm of a wheatstone bridge; changes in temperature of this coil cause an unbalancing of the bridge which applies an emf to a mechanism which controls the current through the furnace.*

The galvanometer G, manufactured by the Weston Electrical Instrument Company, has a period of about 2/3 second and an internal resistance of about 50 ohms. On either side of the contact on the boom is a fixed contact, the boom, of course, being insulated from the coil. On account of sticking of these contacts it is impracticable to allow the galvanometer to actuate directly a relay or switching-mechanism, unless a special device for preventing sticking is adopted.[2] The simplest

[2] See, H. S. Roberts, Journ. Wash. Acad. Sci. *11*, pp. 401-9; 1921.

method is that used by Beaver[3] for minimizing sparking with mercury regulators. The galvanometer-contacts change the grid-potential of an electron-tube, in the plate-circuit of which the relay is connected. The

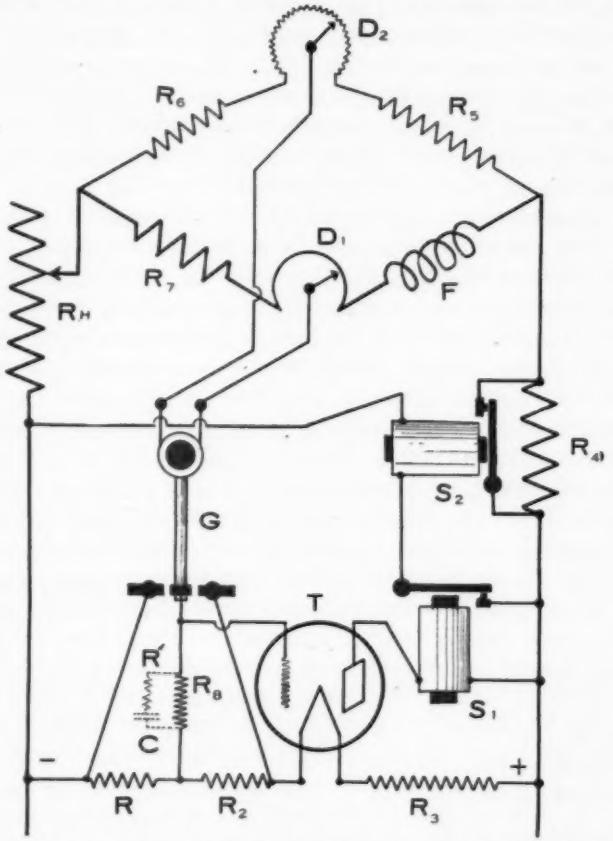


FIG. 2. *Diagram of essential parts and electrical connections of the temperature-regulator. The wheatstone bridge consists of the furnace coil, F; the fixed resistance, R_5, R_6, and R_7; and the variable resistances, D_1 and D_2. The unbalanced emf across the bridge is applied to the galvanometer, G, which, in conjunction with the electron tube, T, and the relays, S_1 and S_2, regulates the heating-current so that the temperature within the furnace remains constant.*

grid-current is so small that the galvanometer-contacts show no tendency to stick, especially if the contacts are made of graphite as sug-

[3] D. J. and J. J. Beaver, Journ. Ind. and Eng. Chem. *15*, pp. 359-61; 1923.

gested by H. S. Roberts.[4] In Fig. 2 T is the electron-tube—preferably a Radiotron, UV 201-A—operated with a little more than 0.2 ampere through the filament. The resistances, R_1, R_2, and R_3 are, respectively, 30, 25, and 445 ohms, and R_1 is joined to the negative side of the line. S_1 is an ordinary telegraph relay wound with No. 40 enameled copper wire, and having a resistance of 10,000 ohms or more, and S_2 is also a telegraph-relay wound with No. 38 wire. R_4 is a coil of such resistance that when short-circuited by the molybdenum contacts of S_2 the current through the furnace-coil F will be increased by about 10 per cent. R_5 is an ordinary "grid-leak" of 3 to 5 megohms.

The operation of the regulator is as follows: the contacts of S_1 and S_2 being open and the galvanometer-boom being in the middle position, the temperature of F will fall and cause the galvanometer-boom to move to the right. When the right-hand contact is closed the grid-potential becomes zero (reckoned from the negative end of the filament) and as a result the plate-current becomes large enough to actuate S_1 and close its contacts, which in turn cause the contacts of S_2 to close, and hence to short-circuit R_4 which increases the current through F and causes its temperature to rise. The boom of G then begins to move to the left and the right-hand contact breaks, thus making the grid-potential -5 volts (below the negative end of the filament), and causing the plate-current to drop to a value which is sufficient to hold the contacts of S_1 closed but which would not close them if they had previously been open. Nothing happens, therefore, until the boom reaches the left-hand contact, when, the grid-potential being now -12 volts, the plate-current becomes so small that S_1 opens, causing S_2 to open and hence decreasing the current through F. The furnace-temperature then begins to fall, the galvanometer swings to the right, breaking the left-hand contact and raising the grid-potential to -5 volts, under which conditions the plate-current is not sufficient to close the contacts of S_1. All parts are now in the original position, and this cycle of operations is repeated over and over again. Each time the boom makes contact on the right the current through F is increased, and each time it makes contact on the left the current is decreased. The *breaking* of the galvanometer-contacts produces no effect. This double-contact system works with more regularity and greater smoothness than the usual single-contact arrangement. The condenser C (0.1 microfarad), although not essential, is useful for preventing chattering of the relays, and the resistance

[4] J.O.S.A. & R.S.I., *6*, p. 972, 1922.

R' (about 10,000 ohms) prevents a surge of current through the galvanometer-contacts due to the charging and discharging of the condenser.

With this regulator the temperature of a platinum-wound furnace at 1400° will automatically remain constant to within 0.1° for hours at a time provided that the thermal insulation of the furnace remains constant. The principal advantages of the method are (1) ease and certainty of operation; (2) freedom from the necessity of placing any part of the regulator within the furnace cavity; (3) high sensitivity; and (4) negligible lag between heater and regulator. This system of automatic control requires that the furnace-coil have an appreciable temperature-coefficient of resistance. It is especially satisfactory with platinum furnaces, but at lower temperatures good results may be expected with heating-elements of nickel, and even of nichrome in spite of its low temperature-coefficient; but it is to be borne in mind that the oxidation of a base-metal resistor connected to this type of regulator will cause a downward drift of the furnace-temperature.

The arrangement shown in Fig. 2 is for furnaces operated with direct current. For alternating current equally satisfactory results are obtained by the addition of a simple vibrating-reed rectifier placed between the bridge and the galvanometer.[5] For further information concerning this and other features of the regulator the various papers mentioned should be consulted.

GEOPHYSICAL LABORATORY,
WASHINGTON, D. C.

Storage Batteries. By George Wood Vinal. viii+402 pages, 156 Figs. Published by John Wiley and Sons Inc., 440 Fourth Ave., New York City. 6×9 inches, cloth $4.50. (22s 6 p, Chapman and Hall, Ltd. London).

This book covers the chemical and physical aspects of storage batteries and their practical applications, and meets a well defined need for an up-to-date standard reference work on storage batteries. It is a scientific treatise suitable for the engineer or those looking for more than first principles but it is not too technical and will prove valuable and interesting reading to any one who wishes to add to his information on the subject.

A good idea of the scope of the book may be obtained from the following list of the most important topics:

Materials and Methods of Construction; The Electrolyte; Theory of Reactions; Capacity; Resistance; Efficiency; Operating Methods and

[5] See H. S. Roberts, J. O. S. A. & R. S. I. *6*, pp. 965-977; 1922.

Equipment; Operating Costs; Dismantling and Assembly; Sources of Trouble; Testing; Modern Applications.

In his discussion, the Author has made good use of the information obtained in recent investigations by the Bureau of Standards with which work he has been identified; also he has had some assistance from engineers of the various storage battery manufacturing companies and other industrial concerns. To these he is obviously indebted for some of the excellent illustrations that are found and for information on manufacturing processes and on modern uses of storage batteries in widely different fields.

The book is well arranged; it is up-to-date, reliable and complete and it avoids that detail of description of particular types of batteries that characterizes repair manuals; for these reasons its recognition as a standard work seems assured. JUSTUS H. BOWER

Photoelectric Emission from a Gold Film Excited by High Frequency Irradiation.—The irradiation in this instance consists of the characteristic radiations (excited by primary x-rays) of nine elements between atomic numbers 34 and 56; they are filtered through 0.75 mm Al and 1.5 mm carbon; the equivalent range of wave lengths, considering the rays to be exclusively x-frequencies, is roughly 1.1A to 0.4A. The current out of the front face of the "extremely thin" gold film, against a P. D. varied from +15 to −15 volt between the gold and a parallel plane electrode 2 mm distant, is measured with a tilted electroscope connected to the film. In every case the current attains a saturation value at about 5 volts accelerating potential; as the potential is lowered and then reversed in sign, the curve bends downward and descends in the familiar photoelectric-current-curve fashion, finally at about −12 volts, reaching a minimum value which however is not zero. This minimum value is supposed to measure the rate of omission of high speed photoelectrons, and it varies strikingly with the irradiation, having a sharp maximum for the rays from element 49 (wave length about 0.52A). The excess of the maximum current over the minimum is supposed to measure the rate of emission of slow electrons, as if these belonged to a definitely demarked class; their amount also varies impressively with the irradiation, and has its maximum at a somewhat greater wavelength. In fact the two curves of current versus wave length can be made to coincide by shifting one through 0.04A relatively to the other, (and multiplying the ordinates of one by a constant factor), as if the slow electrons were in every case excited by rays .04A longer than the fast ones—a curious suggestion of the Compton effect. The high accelerating voltage required to produce saturation of the electron currents is surprising.—[L. Simons, Birkbeck College, London; Phil. Mag. *48*, pp. 250-258; 1924.]

K. K. DARROW

AN OPTICAL METHOD FOR THE STUDY OF SHAFT BEHAVIOR

By B. L. Newkirk

Rotating shafts are subject to certain disturbances that interfere at times seriously with the operation of machinery. Since these disturbances are of small amplitude and occur at speeds too high to be followed by the eye, it is difficult to study them. The shaft being held firmly at the bearings, the disturbance consists generally speaking, of a whirling of the shaft in a bowed condition, or a vibration of the shaft with nodal points at the bearings, one or both of these motions being superposed on the rotation of the shaft.

The diagram of Fig. 1 illustrates an arrangement which has been used successfully in the study of shaft behavior. The diagram shows a model rotor, the shaft S being supported at bearings BB and carrying a disk D and a pointer P. The pointer is illuminated by a beam from a

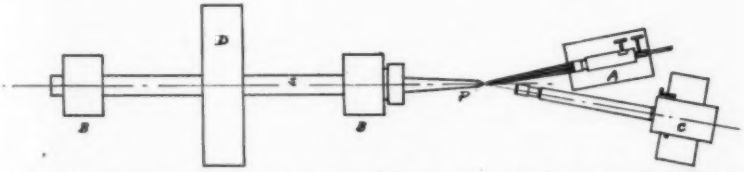


Fig. 1. *Arrangement of motion picture camera (C) and arc lamp (A) for study of the vibration of a model shaft (S) as shown by a pointer (P).*

small projecting lantern A. A motion picture camera C is set to photograph the light reflected from the tip of the pointer. Operation of the motion picture camera gives a succession of short exposures, each one showing the path of the pointer during the time interval of the exposure. In place of the motion picture camera a microscope is sometimes used for visual observation.

It is difficult, of course, to extend the shaft in the form of a pointer with sufficient accuracy to cause the tip to run true. This difficulty is overcome by finishing the pointer when the shaft is running in its own bearings. It is essential that the point be made fine, as nearly spherical at the end as possible, and polished. It is not necessary to blacken the pointer near the tip. In fact the clean polished metal disperses the light

and makes less impression on the photographic plate than the same surface does when coated with lamp black.

The amplitude of shaft vibrations is of the order of .01 to .1 inch at the middle of the shaft, and in order to give patterns large enough for convenient study, the lens barrel of the motion picture camera is lengthened as shown in the diagram to give a magnification of five or six diameters. With arc light illumination and an exposure of about 1/35 of a second, a path one inch in length on the film can be photographed easily.

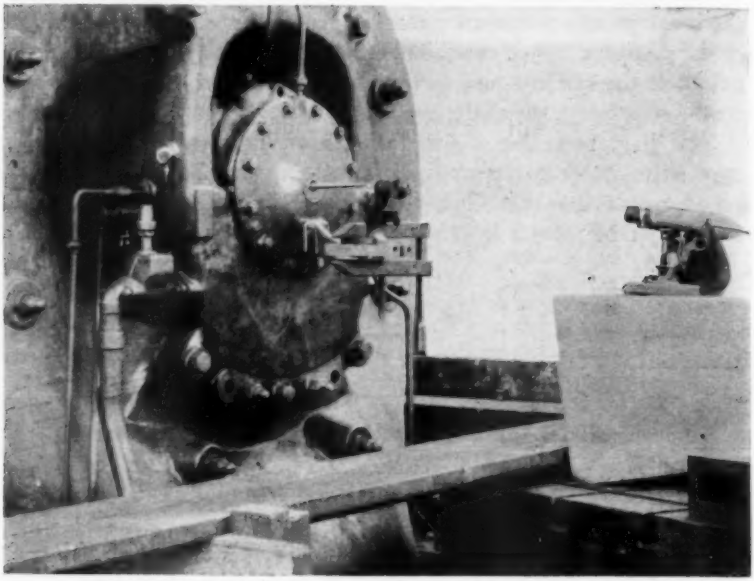


FIG. 2. *Telescope set up for sighting shaft motion of centrifugal compressor.*

It is important to mount the camera or microscope very solidly and to make check runs with the shaft running without disturbance to make sure the patterns are not distorted by motion of the camera or microscope.

Fig. 2 shows an application of this method in studying the shaft behavior of a blast furnace compressor. In this case the pointer was attached to the shaft of the compressor, illuminated by the projecting lantern seen in the picture and observed with a microscope. The path of the pointer appeared as a bright line. When the shaft was whirling

steadily this line was in the form of an oval. Jolts, jars, and irregular vibrations were clearly represented by loops and other irregularities of the bright line. Fig. 3 shows another blast furnace compressor set for observation with a motion picture camera. Fig. 4 shows a record made in studying a model, indicating a gradual building up of a whirling motion of the shaft. The two strips of film should be placed end to end to show the progressive increase of whirling. Fig. 5 shows a double

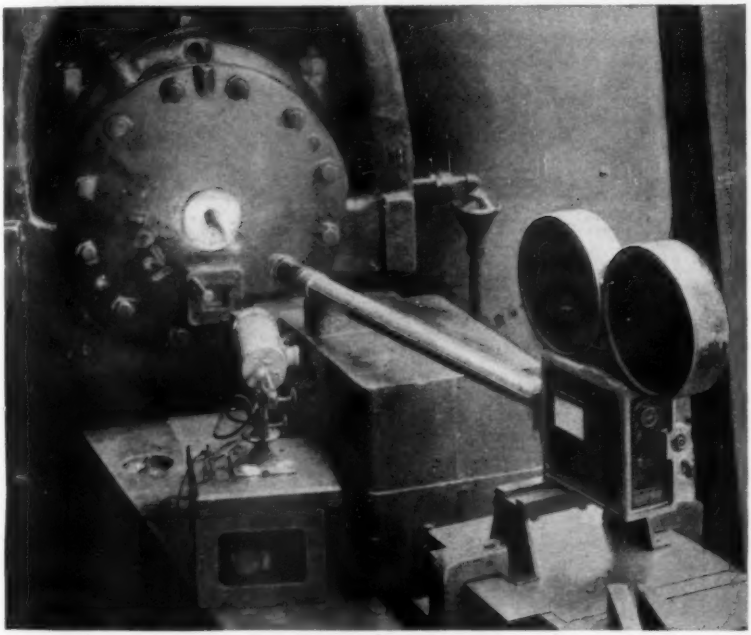


FIG. 3. *Compressor with set up for taking moving pictures of end of shaft.*

exposure made to determine the location of the whirl with reference to the position of the shaft when not whirling. The point in the center of the loop was made with the shaft running steadily. Fig. 6 shows four different patterns due to a superposition of two disturbing influences at different running speeds of the shaft.

The time required for the shaft to traverse any portion of its path may be determined from these photographs roughly as follows: with the shutter wide open and the camera running at normal speed, the time of exposure is about 1/35 of a second. If the photograph shows one

and one-third loops, let us say, completed in this interval it follows that the whirling was at a rate of one and one-third revolutions in 1/35

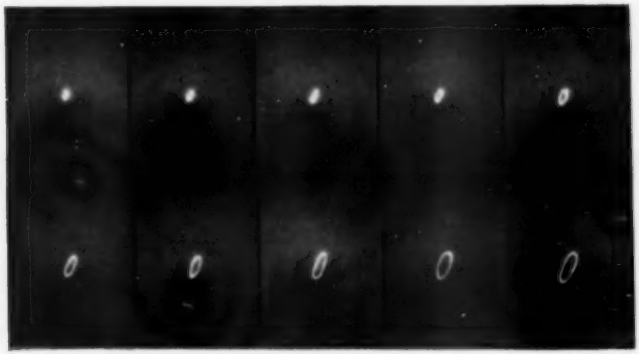


FIG. 4. *Whirling motion building up.*

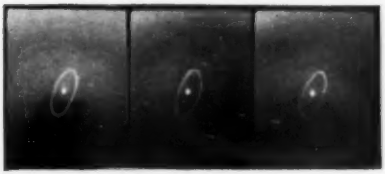


FIG. 5. *Double exposure.*

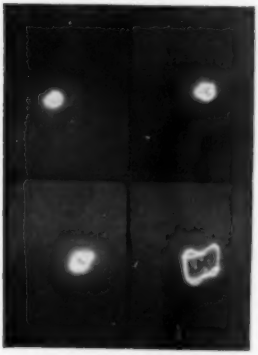


FIG. 6. *Superposed disturbances.*

second, or 2800 revolutions per minute. By running the camera at a different speed similar indications can be obtained for slower or more rapid motions of the shaft.

The magnitude of deflection of the shaft at the middle can be determined approximately by measuring the pattern on the film or in the field of the microscope. Treating the curve of elastic deflection of the shaft, Fig. 7, as though the shaft were bending under gravity due to a load concentrated at the center, the ratio of AB to AC is 2/3. If the stress in the shaft is equivalent to a uniformly distributed load, this ratio is 5/8. Since the determination is only approximate, we may take the average of these values, namely .65. The pointer DE has the direction of the tangent.

FIG. 7. *Relation of pointer deflection (EF) to shaft deflection (AB).*

Setting

l = length of pointer from the bearing center—(DE),

L = length of shaft between bearing centers—(2 DB),

X = Magnifying power of the camera or microscope objective.

d = distance of any point on the photographed or observed pattern from the rest position—(EF),

D = Corresponding deflection of shaft at center—(AB),

then

$$D = .65 \frac{L}{2lX} d.$$

When bearing clearances are large in comparison with the amplitude of motion of the pointer tip, or when the shaft bends into a shape not represented approximately by the common curves of elastic deflection, this formula is not to be relied upon.

This method has proved moderately easy to apply and has given valuable results.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHNECTADY, N. Y.

Photoelectric Effect of various phosphorescent Materials at various Temperatures.—The materials are calcium sulphide with admixtures of bismuth in various amounts, the bismuth being essential to the phosphorescence; zinc sulphide with admixtures of copper, and fluorites with admixtures of rare earths. Electron accelerating potentials of 500-2000 volts were applied; without accelerating potential, no current could be detected; however, the experiments were performed in air at atmospheric pressure. On illuminating the substances with bright unresolved light there was a sudden burst of current ascribed to an electrostatic effect, and then a comparatively constant current of which the mean value, over a period of 30″ after the electrical impulse is taken as the datum. The data, obtained at various temperatures between 18° and 400° C, are confusing and do not suggest any general law. There is no distinct connection between degree of phosphorescence and amount of photoelectric current; the variations of current with temperature which occur (and which are often irreversible) are attributed by Curie to changes in the humidity and conductivity of the salt, affecting the facility of egress of the electrons rather than the rate at which they are excited. [M. Curie; C. R. *179*, pp. 105-108, 1924].

K. K. DARROW

MEASUREMENTS WITH A FABRY AND PEROT ETALON

By John K. Robertson

In teaching Physical Optics to advanced students it is highly desirable that quantitative measurements in the laboratory go hand in hand with formal lectures. Michelson's interference method of measuring the angular diameter of a star becomes vividly real to the student who places a double slit on the table of a spectrometer, and by gradually altering the slit width of the instrument sees and records widths which give zero visibility. Moreover, by means of such experiments he is made to realize the essential unity of mathematical and experimental physics. The mathematical theory of visibility of fringes is no longer a long proof which the student must be able to reproduce but is directly related to physical problems, upon some of which he himself has made measurements. While, therefore, there is little new work in this article, it is felt that no apology is necessary for reporting a series of measurements which can be made conveniently with a Fabry and Perot Etalon by a student in preparation for research, or indeed by any advanced student. At the outset it may be stated that the results submitted in this paper were calculated from measurements on a plate taken and measured by an undergraduate, Mr. H. M. Cave.

The experimental arrangement utilized was extremely simple. Light from a glass mercury vapor lamp, filtered (by means of solutions of potassium dichromate and neodymium chloride) so as to transmit λ 5461, was roughly condensed on the Etalon, and the resulting system of fringes photographed with the camera attachment of a Hilger constant deviation spectroscope. Diameters of successive rings, as given in the second column of Table 1, were measured with a Gaertner travelling microscope reading to 0.005 mm.

To make clear the use to which these measured diameters may be put, it is necessary very briefly to state the simple theory of the Etalon. If p_n = the order of interference at the n^{th} ring from the center, t = thickness between the half-silvered surfaces, and x_n = the *angular* diameter of the n^{th} ring, then

$$p_n = \frac{2t}{\lambda} \cos \frac{x_n}{2}, \qquad (1)$$

from which, since x_n in practise is a small angle,

$$p_n = \frac{2t}{\lambda}\left(1-\frac{x_n^2}{8}\right) \tag{2}$$

or, if d_n = *linear* diameter of n^{th} ring and
f = focal length of the optical system forming rings on the plate,

$$p_n = \frac{2t}{\lambda}\left(1-\frac{d_n^2}{8f^2}\right) \tag{3}$$

TABLE 1

Ring Number	Diameter in mm	d_n^2	$d_n^2-d_{n-1}^2$
2	8.247	67.99	
3	11.500	132.25	64.25
4	14.018	196.50	64.25
5	16.149	260.79	64.29
6	18.018	324.65	63.86
7	19.696	387.93	63.28
8	21.268	452.33	64.40
9	22.726	516.47	64.14
		Mean	64.07

MEASUREMENTS

I. CONSTANCY OF $d_{n+1}^2-d_n^2$

From equation 3 it follows at once that, as long as the angular diameter of a ring is sufficiently small to permit of the use of equation 2, $d_{n+1}^2-d_n^2$ must be constant. The agreement of theory with observation will be seen by a glance at the numbers in the fourth column of Table 1.

II. VALUE OF $\Delta\lambda$, THE CHANGE IN WAVE-LENGTH WHICH CORRESPONDS TO A SHIFT OF ONE FRINGE

If d_n and d_n' represent the linear diameters of the n^{th} rings for wave-lengths λ and $\lambda+d\lambda$, it is easy to deduce from equation 3 that

$$d\lambda = \lambda\frac{d_n^2-d'^2_n}{8f^2} \tag{4}$$

The value of $\Delta\lambda$, therefore, is equal to the value of $d\lambda$ obtained when $d_n^2-d_n'^2$ = the difference in the squares of diameters of two successive rings.

$$\therefore \Delta\lambda = \lambda\times\frac{64.07}{8f^2}$$

$$= \frac{5460.74 \times 64.07}{8 \times (543)^2}$$

$$= 0.148\ A.U.,$$

since the measured focal length of objective of camera = 543 mm.

Should a line with a satellite be photographed, equation 4 at once permits of calculation of wave-length of satellite.

III. THICKNESS OF ETALON

From equation 2 it may readily be shown that

$$t = \frac{4\lambda(n-k)}{x_n^2 - x_k^2} \tag{5}$$

where n and k are the numbers of any two rings. If we make use of the diameters of the second, third and fourth rings, we obtain the values for t given in Table 2.

TABLE 2.

k	n	t
2	3	10.02 mm
2	4	10.02 mm
3	4	10.02 mm

The value of t obtained in this way differs by about one per cent from the thickness supplied by the manufacturer, 10.135 mm. While an alteration in the latter is possible, it is not probable, and in the opinion of the writer, the error is to be found in the difficulty of obtaining an accurate value of f. In this connection it may be noted, as explained in Section V, that it is possible to obtain f by calculation.

IV. FRACTIONAL PART OF ORDER OF INTERFERENCE AT THE CENTER

In general, the order of interference at the center of a ring system, that is, the order corresponding to the thickness $2t$ is given not by an integral number but by $p_1 + \alpha$, where p_1 = the order of the first ring and α = a proper fraction. In the standard use of the Etalon for the comparison of wave-lengths, the value of α must be obtained for each wave-length as accurately as possible. By combining equation 2 with the relation

$$p_1 + \alpha = \frac{2t}{\lambda}$$

it is easy to show that

$$a = \frac{p_1 x_1^2}{8}, \tag{6}$$

the expression which is generally used to obtain its numerical value. When obtained in this way, obviously its value depends on the magnitude of three quantities, (1) p_1, (2) the linear diameter of the first ring, (3) the focal length of the focussing arrangement. Now, while p_1 can be obtained by a method due to the late Lord Rayleigh[1] without any uncertainty, difficulty may be experienced in measuring the required focal length with sufficient accuracy. There is much, therefore, to be said for obtaining a by means of a method similar to one in use at Mount Wilson Observatory. If we again make use of the fundamental equation, we may readily prove that

$$a = \frac{(n-1)d_k^2 - (k-1)d_n^2}{d_n^2 - d_k^2}, \tag{7}$$

where n and k are numbers of any two rings.

This expression, therefore, provides a means of obtaining a which depends solely on the linear dimensions of diameters. Values of a obtained in this way are given in Table 3.

TABLE 3

k	n	a
2	3	0.0581
2	4	0.0581
2	5	0.0579
3	4	0.0584
3	5	0.0577
4	5	0.0565

Mean value = 0.0578.

V. CALCULATION OF f

If we make the reasonable assumption that the value of t supplied by the manufacturer is correct to the order of figures given, it is possible to calculate f and thus remove the necessity of obtaining its value by experiment. This is done by making use of the relation

$$p_1 = \frac{8f^2}{d_n^2 - d_k^2} + k - 1 \tag{8}$$

which again may readily be deduced from the fundamental equation.

[1] Rayleigh, Phil. Mag., *11*, p. 685; 1906.

In very precise work p_1 would be obtained by the Rayleigh method, a process too long and laborious to be suitable for laboratory instruction. From the supplied instrument thickness, however, its value may be obtained directly to an accuracy of the order of one-tenth per cent. Thus, taking $t = 10.135$ mm. and $\lambda = 5460.74$ A.U. we have

$$p_1 = \frac{2 \times 10.135}{5460.74} \times 10^8$$
$$= 37119.$$

Substituting this value in equation 8, with values of n and k chosen from 2, 3 and 4, we obtain for f the value 546 mm. as against the experimental value 543 mm.

We may note that if we take this value of f, and calculate α by use of equation 6, (modified so as to apply to the second ring) we obtain a value $= 0.0581$, as compared with 0.0578 given in Section IV.

QUEEN'S UNIVERSITY,
KINGSTON, CANADA.
MAY 20, 1924.

Theoretical Speculations on the Photoelectric Effect.—Kirchner tabulates observed values of threshold wave length for twelve solid elements, and compares them with the resonance potentials of the elements (translating wave lengths into potentials by the quantum relation); he also tabulates wave lengths of maximum photosensitiveness for the alkali metals and two others, and compares them with the differences between ionization and resonance potentials. The idea is that photoelectric action begins at the frequency just adequate to extract electrons from a free atom of the substance by three or more consecutive absorptions, and attains a maximum at the frequency just adequate to extract electrons by two consecutive absorptions. (It happens that in all the seven cases of the latter tabulation the ionization potential exceeds double the resonance potential, so that these two critical frequencies do not coincide.) It is difficult to accept a theory involving the assumption that the atoms in question behave like free atoms, true though it be that the photo-effect seems to be localized near the surface of the substance; one may object that the smoothly rounded selective effect maximum does not at all suggest a sudden sharp increase, and that it depends on the orientation of the solid surface relatively to the polarized electric vector; and the agreement between theory and experiment, as brought out by Kirchner, is scarcely strong enough to overcome the objection. There are some remarkably close coincidences, but the discordances in the instances of Li, Na, and K are equally striking. [F. Kirchner, Munich; Phys. ZS. *25*, pp. 303-306; 1924.]

K. K. DARROW

Impacts of positive Ions against Platinum.—Positive ions from a heated film of aluminium phosphate on platinum were projected against a platinum cathode with voltages varying from 75 to 600 and the current into the cathode plotted as function of voltage. The question at issue is whether part of the current consists of efflux of electrons, and if so, whether the efflux begins abruptly at some critical value of the energy of the bombarding ions. To eliminate the variation of the rate of emission of positive ions from the film with the applied voltage, Badareu conceived the device of subdividing the potential-drop, applying a constant voltage 75 between the film and a grid in front of it, and the variable additional voltage between the grid and the cathode. Yet the current into the cathode increased smoothly and linearly with voltage over the entire range with no sign of inflection or corner in the curve. [E. Badareu, Czernowitz; Phys. ZS. *25*, pp. 137-140, 1924.]

K. K. DARROW

A METHOD FOR MAPPING MAGNETIC VECTOR FIELDS

By Paul Kirkpatrick

The construction of maps of magnetic fields is an exercise common to nearly all general courses in experimental physics and one frequently used also in courses in electrical measurements. The most popular methods in use depend upon observation of chains of magnetized iron filings or upon explorations with a moving compass needle. These methods possess the merit of simplicity but in other regards leave somewhat to be desired. Maps made in either of these ways present not simply the field in question but a combination of this field with that of the earth. Errors committed in locating points by the second method are cumulative, leading to unsymmetrical curves when the orientation would require symmetry. A more important matter is that these methods determine only the direction of the magnetic field whereas of course its real nature is vectorial and variations in magnitude should be equally as interesting, if determinable, as the direction variations.

The method to be described permits the determination through one series of operations of the direction and magnitude of the magnetic field vector. The magnitude is stated in terms of the local field of the earth or, when this is known, in gausses. The final plot shows the true lines of the poles under investigation, uninfluenced by the terrestrial field.

In studying the field of, for example, a bar magnet the magnet is mounted upon a horizontal board capable of rotation about a central vertical post fixed in a table top. The post also serves to support, just above the board, a compass needle and its angular scale. Fig. 1 shows the arrangement from above. The direction of the field of the magnet at the axis of the board may be known by noting the orientation of the board for which the needle lies in the direction of the horizontal component of the earth's field. If from such a position the board be rotated through ninety degrees the fields of the earth and of the magnet will be normal to each other at the compass needle and hence the tangent of the angle of deflection gives the ratio of the field strengths.

As is shown by the figure the table top is provided at one side with a straight-edged rail fixed just above the level of the board to serve as guide for a T square. With drawing paper affixed to the board and the

compass mounted at the center, as described, the following preliminary adjustments require to be carried out. The compass box is turned until the zero line of its scale corresponds in direction with an edge of the T square, the same being in contact with its guide. The box is fixed in this position in rigid connection with the table. The table as a whole is rotated in the horizontal plane until the compass needle, swinging freely in the earth's field, rests at the zero of its scale.

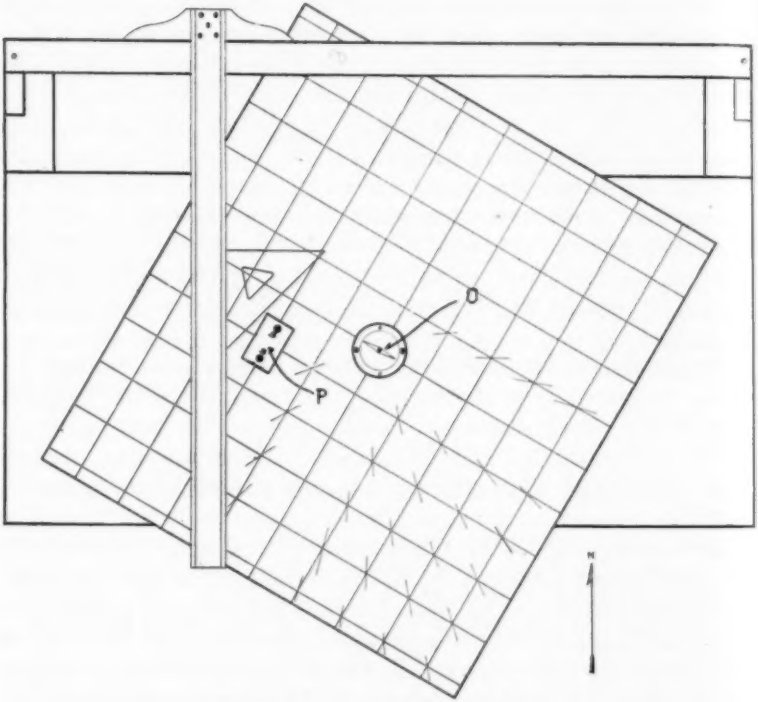


FIG. 1. *Mapping table seen from above.*

Observations may now be taken which shall give for a random point P on the drawing board the magnitude and direction of the field of the bar magnet considered as fixed at O, the center of the board. This location being occupied by the compass the magnet is never actually placed here. Its real position is P and its azimuthal orientation is such as to differ by 180° from that of its virtual position at O. It is obvious that the hypothetical field at P is identical in magnitude and direction

with the real field at O, though the two are opposite in sense. All the measurements to be described are measurements of the real field at O, the magnet being placed successively at a number of points P, chosen so as to cover the board uniformly.

With the magnet properly placed at P the compass needle will in general be deflected from the north-south line. By rotating the board, and hence the magnet, this deflection is reduced to zero. The fields of earth and magnet, separately considered, are now coincident and

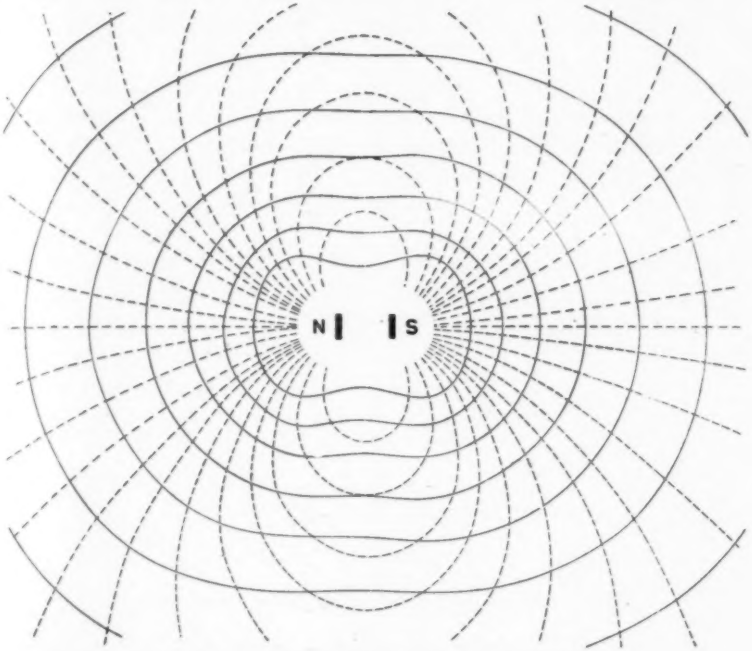


FIG. 2. *Intensity and direction of the field of two unlike poles.*

their common direction is that of an edge of the T square. The square is brought across the board at any convenient place and a celluloid right triangle fitted against it. The square is lifted and the board turned through ninety degrees or until the square aligns with another leg of the triangle. The deflection of the needle under the crossed fields is observed and recorded on the paper near P. The magnet may be removed and, using the square and triangle, a pencil line drawn through P recording the field direction. The observations described in this

paragraph are found to consume about half a minute, deflections being read to the nearest third of a degree.

The magnet is moved to a new point and the observations repeated. It has been found convenient to rule the paper into squares by parallel lines ten centimeters apart and to place the magnet at the intersections, orienting it with respect to the lines.

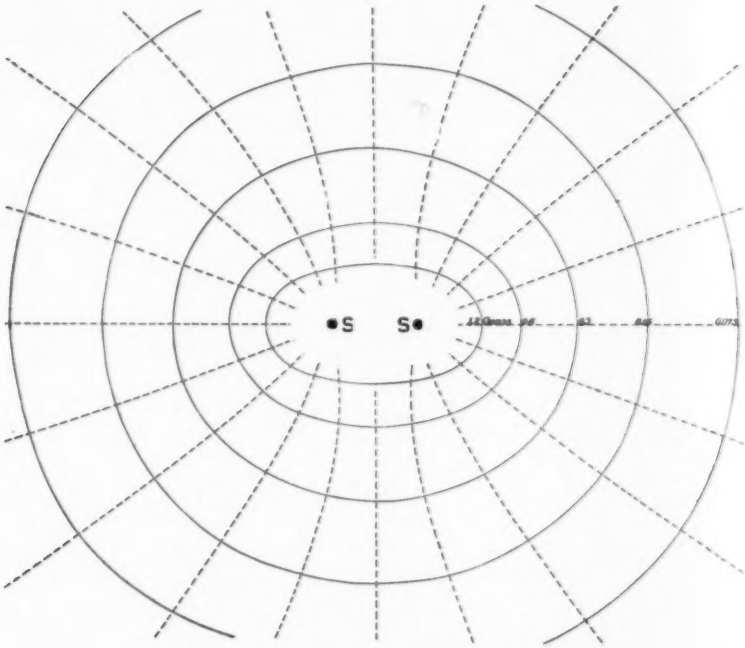


FIG. 3. *Intensity and direction of the field of two like poles.*

After observing at a sufficient number of locations to indicate the course of the lines of force the tangents of the recorded angles are obtained from a table and jotted down at the intersections. If desired the tangents may be multiplied by the local value of H, the product giving the magnet field strength in gausses. The variations of the field strength may perhaps be best exhibited by a series of isodynamic lines and these may now be sketched in by investigation or with the help of roughly plotted curves.

The accompanying figures show two typical maps. Fig. 2 shows the field of two equal dissimilar poles. A U-shaped magnet was used,

being mounted with its plane vertical and poles downward. The solid lines are loci of equal field intensity and the value on any one of them is double that of the next one outward. Although the field exhibits a dependable fourfold symmetry this circumstance was ignored in the mapping and all quadrants were covered independently. The map is instructive in its display of the manner in which intensity decreases with distance from the poles. Plotting intensity against the reciprocal of the cube of distance along either axis of symmetry an excellent straight line results. It is readily seen that at a given distance the intensity on the line containing the poles is double the intensity on the normal.

Fig. 3 depicts the field of two similar poles. Two like ring-stand rods were magnetized together in a helix and then withdrawn and mounted vertically on a wooden base. The numbers on the isodynamic lines in this figure denote field strengths in gausses. Here, as in the preceding figure, the field strength at any isodynamic is double the value at the next one outward.

The original plates were 100 by 110 centimeters. They indicate that the assembly is well adapted to laboratory use, having been executed by a sophomore student, Mr. C. H. Kang, to whom the writer gladly acknowledges his indebtedness.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF HAWAII,
HONOLULU, T. H.

Year Book of The American Engineering Standards Committee.—There now exists a wide-spread interest and activity in industrial standardization according to the new Year Book of the American Engineering Standards Committee.

The work of the Committee is indicative of the growth of the movement as a whole. One hundred and fifty-two projects have been completed, or are under way, and in these various projects two hundred and thirty-five national organizations, technical, industrial, governmental, are officially cooperating through accredited representatives. The number of the individuals serving under various sectional committees of the different projects is nearly 1100.

Of the projects which have reached an official status, 31 have to do with civil engineering and the building trades; 25 with mechanical engineering; 15 with electrical engineering; 4 with automotive subjects; 11 with transport; one with ships and their machinery; 14 with ferrous metals; 15 with non-ferrous metals; 12 with chemical subjects; 2 with textiles; 5 with mining; 5 with the wood industry; 1 with the paper and pulp industry; and 11 projects with topics of a miscellaneous or general character. Among the projects are: Specifications and Tests for Portland Cement; Code of Lighting Factories, Mills, and other Work Places; Code for Lighting of School Buildings; Electrical Properties of Aluminum; Colors for Traffic Signals; Specifications for Solder Metal; Dimensional Standards for Motion Pictures; Terminal Markings for Electrical Apparatus; etc.

In order to meet the demands made upon it by industry, and to supply the needs of the various working technical standardization committees, the American Engineering Standards Committee has greatly broadened its information services, and has added an Engineer Translator to its staff for this purpose. In this way, complete information is made available to sustaining members, trade and technical associations and other inquirers on standardization activities in foreign countries, as well as in the United States.

One of the most striking developments of the stndardization movement is the increasingly important role which trade associations are playing in it. More than 140 national trade associations are officially participating in standardization projects under the auspices of the American Engineering Standards Committee. That standardization is a legitimate and constructive activity for associations is everywhere recognized, and explicitly so by a recent decree of the U. S. District Court at Columbus, Ohio. More and more trade associations, no matter for what purpose they may have originally been organized are going into technical work, and it is natural if not inevitable, that both trade and technical associations should eventually concern themselves to an increasing extent with standardization activities. This tendency is becoming more marked on every hand.

The Year Book may be secured by addressing the American Engineering Standards Committee, 29 West 39th Street, New York City.

A CONVENIENT APPARATUS FOR MAKING SPIRAL HEATERS, FILAMENTS AND SPRINGS

By Richard Hamer

The apparatus described below enables one to make neat spiral filaments, heaters and springs in a very rapid manner more or less automatically instead of the tiresome way of making them by hand. The writer believes there are some who would like to learn of such a method.

For the usual size of filament or spring $\frac{3}{8}$ inch diameter a piece of brass rod 2 inches long and $\frac{3}{8}$ inch diameter (see Fig. 1) should be cut off and machined with a very smooth face *A*. One hole *BC* should be drilled with a number 57 drill centrally to a depth of about 1 inch and one with a number 60 off from it at *C* obliquely to the outside about $\frac{1}{2}$ inch down (see *DC*). Then two holes *E* and *F* should be drilled with a number 50 drill as shown and tapped for two machine screws number 2-56. One of these *E* should go down to the central hole to clamp the axial spindle. It is convenient to use a large size steel sewing needle for this to fit the central hole size 57 rather snugly. It was found that the holes *BC* and *DC* should not merge absolutely at *C* but care must also be taken not to have the opening too wide. It should be wide enough for the needle and the wire on one side.

For wire that is very brittle it is advisable to carefully file the opening of *DC* at *C* just a little so that the bend for the wire does not form too sharp an angle.

Another portion of the brass rod $\frac{3}{8}$ inch diameter should be cut off and drilled centrally with the number 57 drill. The face *G* should be finished off very smoothly. In fact it pays to do the same with both ends. It also is to be drilled at 0 with a number 50 drill for a machine screw number 2-56 which must go down to the central hole.

In order to wind a spiral it is advisable to clamp the longer portion *MA* in the chuck in the lathe after inserting one end of the tungsten wire which should be about 6 inches in length in the oblique hole and clamping it tight by means of the machine screw *E*. The outer end of the wire should now be turned over and around the needle spindle and then firmly held parallel to the face *A*. The other piece of brass *GN* is now placed over the needle and pushed up tight against the face *A* with

the wire between. The machine screw helps a little to clamp it there, but it is advisable to use another chuck held in the tail stock and clamped in position. The needle spindle should however be free to rotate inside the chuck as shown.

The lathe should now be turned slowly by hand and the wire fed into the gap as uniformly as possible under a certain amount of tension. After the desired number of turns or the gap AG being filled the smaller piece GN is first released and then the screws E and F. The result should be a neat looking spiral with the turns not in contact with one another.

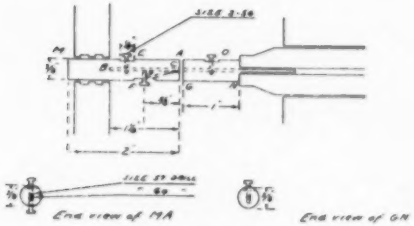


FIG. 1

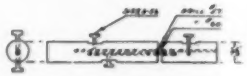


FIG. 2

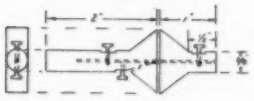


FIG. 3

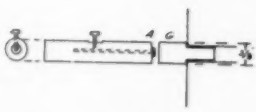


FIG. 4

Though the wire touches at each turn while it is being wound it will generally separate evenly and sufficiently for insulation purposes as it unwinds a little on removal. Sometimes it is wise to unwind the wire a little, forcibly pushing on the wire tangentially while it is still held firmly in the apparatus.

The above apparatus will be found to work very well for wire of 14 mil or over in diameter. For smaller wire I believe it would be best to make it of steel so that the faces A and G could be given a very hard and very smooth finish. Brass seems to grip finer wire and hold it out of its proper radial distance. It then bends angularly at intervals and the turns remain in contact at those places when the spiral is removed. By exerting considerable tension one can prevent this to a certain extent but the wire does not separate nicely afterwards as it does with larger diameters. I have tried winding the fine wire along with

cotton thread or nickel wire clamped and threaded through the oblique hole. After the thread is removed the turns are usually free. When wire is used it is sometimes difficult to separate the two spirals.

For non-inductively wound spirals it is necessary to make a new apparatus with an extra shaft close to the central shaft and of course excentric to it. A finer steel needle can be used to fill this shaft and the corresponding hole in the part *GN*. The needle would need to be clamped in the part *MA*. See Fig. 2.

The design shown in Fig. 2 could also be used for ordinary spirals if the excentric needle is omitted and the excentric hole used for the wire. It would need to be drilled very close to the central one.

For larger spirals it is only necessary to use larger diameter material for the two pieces or else just leave the face plate ends *A* and *G* large enough and reduce the diameter conically as shown in Fig. 3.

A lathe is not entirely necessary if the machine screws *E* and *O* are made real tight on the needle. With a little practice good spirals can be made by rolling the brass sections slowly between the fingers of the left hand while the proper tension is applied with the right hand.

Another type of apparatus which perhaps is simpler and easier to make is shown in Fig. 4. In this there is just one central hole made by a size 57 drill with another radial one about 1 inch down crossing it made by a size 50 drill and tapped for a machine screw 2-56.

The end face plate *A* should be left with a small projecting portion short in length and of small diameter. This should be sawn across along a diameter by a fine saw down to the level of the face plate.

The wire in this case is pushed down the central hole and clamped. Then it is bent over into the sawn groove and wound around after the other face plate is held against it firmly by the other chuck in the clamped tail stock. Dimensions can be made to suit the particular size of wire used for the spiral heater, filament or spring.

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Kristalle und Röntgenstrahlen. By Professor P. P. Ewald. 327 pp. Published by Julius Springer, 1923.

Professor Ewald's book, "Kristalle und Röntgenstrahlen," has the same relation to the crystal work as it stands today that the Braggs' original book, "X-rays and Crystal Structure" has to the time of its publication. Professor Ewald gives a very complete resumé of the present status of the study of crystals by means of X-rays. But Professor Ewald has done more than to write a mere resumé. He has worked over the material so that the presentation is distinctly his own. It is a pleasure to read a book, written on a purely scientific subject, in which the personality of the author creeps in from time to time, as for instance in the second chapter on the fundamental principles of crystallography, in the fifth chapter on the scattering and interference of X-rays, and in the ninth chapter on the reflections from zone-axes by the Laue method.

The book opens with a general discussion of the theory of atoms, leading up to a discussion of the crystal state in the second and third chapters. The fourth chapter deals with interference phenomena. The half-tone pictures used to illustrate waves and their interference are exceptionally good. The subject of interference is taken up in the usual way using both two and three dimensional gratings. Chapter six serves as an introduction to the various methods of crystal analysis, taking up in succession the outstanding features of the Laue, Bragg, and powder methods. The Bragg method is continued from chapter six into chapters seven and eight. The Laue method is further treated in the ninth and tenth chapters and in Note III. This treatment would have been more coherent if Note III had been incorporated in chapter ten.

The powder method of Debye-Scherrer and Hull is taken up in the eleventh chapter and in Note IV. Here again, one might wish that the material in the note had been given in chapter eleven itself. It is characteristic of the thoroughness with which the author has gone into the whole subject, that the graph shown in Fig. 176 of Note IV is not to be found in the original article to which reference is made, but has been recalculated by the author so as to apply to an entirely different system of nomenclature. Although it is not distinctly brought out in the title of the graph, it gives, on a logarithmic scale, the diffraction pattern to be expected for the simplest possible rhombohedra of various solid angles. When the angle equals 90°, the graph, therefore, shows the pattern of a simple cube. At this point, in terms of the original nomenclature, in the hexagonal system, the axial ratio is 1.225. The rhombohedral lattice also meets the cubic system at two other axial ratios: 2.45 for the face-centered cube and .612 for the body-centered cube.

It is an interesting commentary on the state of X-ray work in Germany that the author is astonished at the amount of exposure used

in this country in the powder method. The exposure of 15 hours which excites the wonder of the author is very short compared with many that have been taken in this country. This is partly because we use greater distances for the sake of increased accuracy, and partly because the intensity of lines in the powder patterns of certain substances falls off very rapidly as the "angle of reflection" is increased. For this reason the writer has sometimes given exposure of 180 hours or more at 25 M.A. in order to bring out certain lines corresponding to small interplanar distances. Such an exposure would be, of course, unthinkable to one used only to the obsolete type of X-ray tubes illustrated in the book.

Chapter twelve consists of three parts which deal respectively with the so-called "complete" diffraction patterns of Seeman and Schiebod, the patterns obtained by diffraction due to an arrangement of atoms in the molecule (fiber-structure) rather than to true crystals, and the patterns obtained from distorted crystals. Chapter thirteen gives examples of the various type of crystal structure, each with a diagram to make the structure clear. All these diagrams are exceedingly well done. Many of the structures are further illustrated by half-tone pictures. Chapters fourteen and fifteen deal with symmetry considerations, isomorphous substances, ionic compounds, radii of ions, and crystals of mixed composition, i.e., solid solutions of variable composition, (so often mis-translated "mixed crystals"). The structures of complex molecules are given in chapter sixteen. Chapter seventeen takes up the applications of crystal structure data to various theoretical problems, (1) general laws and particular constans, (2) elasticity, (3) piezo-electric effect, (4) resonance frequencies, (5) specific heat, (6) dispersion, (7) double refraction, (8) reststrahlen (9) density, and (10) compressibility. The treatment of these points is quite fragmentary. One could wish that the author could have found space for a more detailed review of the present state of some of these topics.

The book closes with a critical summary of the crystal data so far published. This summary is probably the most complete in print today. In only one case has the writer noticed an unwarranted refusal to accept the accuracy of published data. That is in the case of graphite, where he publishes the interpretation of Debye and Scherrer, bus dismisses the interpretation of Hull with the mere statement "falsche Structurbestimmung." The two structures are each composed of pairs of equilateral triangles, the members of the pairs being directly above each other. Debye and Scherrer claim three interpenetrating lattices of this sort, while Hull's data require two. This makes the Debye-Scherrer solution a rhombohedron, while the Hull solution is a "hexagonal close-packed" lattice. The two solutions require different vertical spacings of the members of the pairs. Now Hull's data, taken on several samples of graphite, both natural and artificial, agree completely with his interpretation but not with that of Debye and Scherrer. Debye and Scherrer's data fit Professor Ewald's graph, but

not that for Hull's interpretation. Under the circumstances, it would seem that the author should have included both structures in his summary. It is inevitable that a summary of data on such a live subject as crystal analysis should be out of date as soon as it is published. But Professor Ewald's summary is beyond question, the most complete in existance up to about March, 1923.

The book itself is very easy to read. It speaks volumes for Professor Ewald's grasp of his subject that his book is easy to translate into an uninflected language like ours.

WHEELER P. DAVEY

A SUBSTITUTE FOR LIQUID AIR IN LOW PRESSURE WORK

By Franklin E. Poindexter

1. Introduction

The mercury diffusion pumps which are used to produce low gas pressures have the disadvantage that, unless there is some sort of a trap between the pump and the part of the apparatus being evacuated, the lowest pressure which can be obtained is of the order of that of mercury vapor at the temperature of the room. The trap is usually a glass tube immersed in liquid air. The trap operates by the condensation of the mercury vapor on the sides of the tube, hence it would seem that a metal which would form an amalgam with the mercury vapor might have same effect as the cold glass surface. It has been found that a trap containing sodium, but not immersed in liquid air, reduces the pressure to practically the same amount as can be obtained by a liquid air trap. The metals cadmium, zinc and gold were also tried in the trap. Cadmium was found to reduce the pressure but not so well as sodium; while zinc and gold produced only very slight reductions in pressure and were therefore rejected after preliminary tests.

2. Apparatus

A diagram of the apparatus is shown in Fig. 1. A Buckley[1] ionization gauge, I, using the internal control connection of Dushman[2] was connected to the trap, T, through a wide tube approximately 2 cm diameter. The trap was connected to the mercury reservoir, R, through 1 meter of tubing, M, of .6 cm diameter. The reservoir was connected directly to a MacLeod gauge and diffusion pump not shown in the diagram. The electron current from the filament, F, to the plate, P, was measured by a galvanometer, G_1, shunted with a universal shunt and having a sensitivity with the shunt at 1 of 2.66×10^{-8} amperes per scale division. The ion current to the grid, G, was measured by a critically damped galvanometer, G_2, whose sensitivity was 8×10^{-10} amperes per scale division. The filament current was controlled by means of parallel rheostats. The plate, P, was held at a positive potential of 250 volts and the grid, G, at a negative potential of 24 volts throughout all of the experiments, storage batteries being used for all potentials. All of the

[1] Proc. Nat. Acad. Sciences, *2*, p. 683; 1916.
[2] Phys. Rev., *17*, p. 7; 1921.

glass parts were of pyrex glass with fused glass joints and were outgassed by heating directly in a gas flame for several hours with a large hand burner. The plate, P, which was of nickel and in the form of a cylinder 2 cm long by 1 cm in diameter, was outgassed by heating to a bright red heat with electron bombardment at 10,000 volts from a transformer. The grid, G, was a double spiral 7 mil tungsten filament and was outgassed by heating white hot with a current. The filament was approximately 5 cm long and of 7 mil tungsten supported in the middle by a tungsten wire stuck into the glass between the leads.

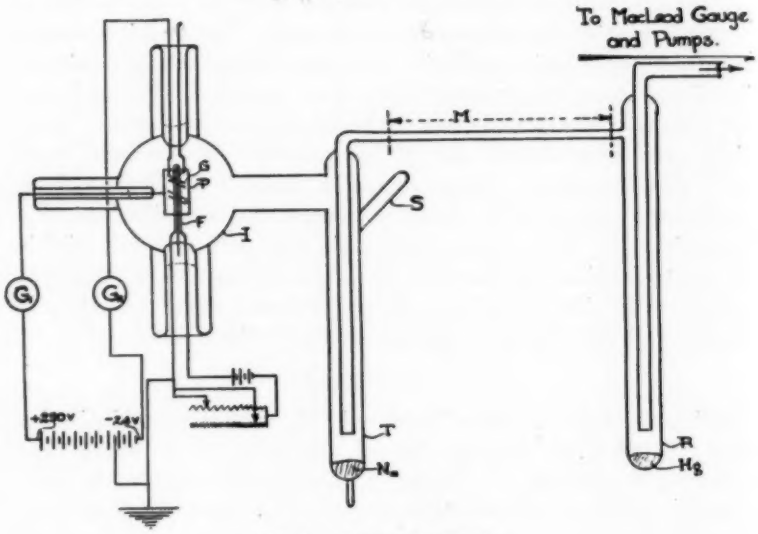

FIG. 1. *Diagram of apparatus.*

The cadmium and zinc were introduced through a side tube, S, and evaporated up from the bottom of the trap by means of a flame. The gold was plated on to a nickel cylinder and introduced by cutting the trap and resealing. A tube containing sodium which had been purified was sealed to the bottom of the trap, T, Fig. 1, and two or three grams driven in with a bunsen flame. In order to carry out this purification, the sodium was first placed in a tube of the form shown in Fig. 2, which was connected to an auxiliary set of pumps, and carefully distilled in vacuo to successive compartments. The compartment adjacent to the pumps Fig. 2 being the one which was finally sealed to the trap, T, of Fig. 1.

3. DATA AND RESULTS

The data were obtained in runs of four to eight hours and repeated on successive days until the results were reproducible. Fig. 3 shows the readings obtained on two successive days with sodium in the trap. It will be noticed that the ordinates of curve 2 taken on the second day

FIG. 2. *Tube used for purification of sodium.*

are uniformly shorter than those of curve 1. This decrease in the ion current reached a minimum on the thirteenth day after the sodium was introduced and will be discussed at length under "Gas Effect."

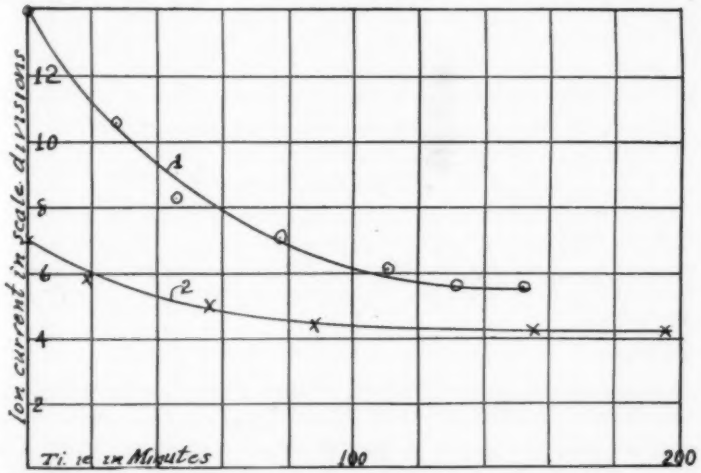

FIG. 3. *Sample runs of data. Curve 1 was obtained two days after the introduction of the sodium into the trap. Curve 2 was obtained two days later than curve 1.*

Effect of Admitting Air.

Air, dried by passing through $CaCl_2$ and P_2O_5, was admitted to the apparatus for ten minutes at atmospheric pressure. The effectiveness of the sodium trap was not altered to a detectable extent. Dry air was again admitted, the meter length of tubing, *M*, cut out and the sodium trap sealed directly to the mercury reservoir. The Na was exposed to the air at atmospheric pressure for twenty minutes together with the moisture incidental to the sealing of the joint. Again there was no

detectable increase in the mercury vapor pressure which passed the trap.

Effect of Lessening the Distance Between the Trap and Hg Reservoir.

The cutting out of the meter of tubing between Hg reservoir and Na trap had no effect.

Effect of Forming New Surface of Na in Trap.

Forming a new surface in the trap had no effect.

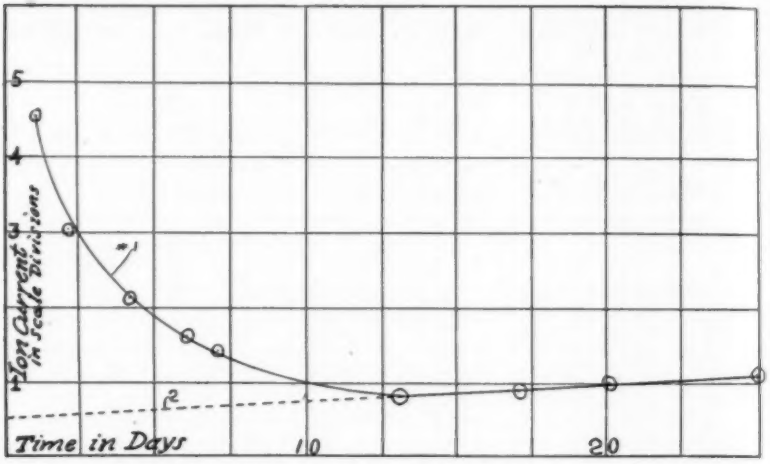

FIG. 4. *Curve for the whole series of readings with sodium in the trap. Part 1 shows the decrease in the "Gas effect." Part 2 shows the straight line increase in the mercury vapor pressure passing the trap.*

Gas Effect.

After the sodium was introduced on June 10th, the lowest reading—an ion current of .8 division—was obtained on June 23d, the Na having been continuously exposed to the Hg vapor. The rate of fall of the pressure with the sodium trap on is shown in the curve No. 1, Fig. 4. The steady decrease was probably due to two causes. First, the Na was not as thoroughly outgassed before the introduction as it might have been and consequently gave off a steadily decreasing stream of gas. This view is supported by the fact that upon standing over night following the first day's run the MacLeod gauge reading showed a pressure of .002 mm of gas. The second morning the pressure was .000135 mm, the third it was .000034 mm and was less than .00002 mm each morning thereafter showing that the Na had become practically free of gas. Second, the glass wall of the ionization gauge—which was quite hot while the filament was burning steadily—gave out a diminishing

stream of gas. This was verified in two ways, first, by allowing the gauge to cool and noting the drop in the reading and, second, by closing the mercury gate leading to the pumps and thus allowing the pressure to build up. On June 11th the ionization current became steady at 7 scale divisions. The filament was shut off for five minutes. The ion current dropped 2.5 divisions. This drop upon allowing the gauge to cool gradually diminished till on June 27th it was only .1 division. On June 16th the gate leading to the pumps was closed for twenty minutes with the filament burning steadily. The ion current rose 18 divisions compared to a rise of 3.3 divisions in the same time on June 27th. One is therefore justified in concluding that this gas effect is due to a gradual liberation of gases from the walls of the gauge under the influence of the heat from the filament. It is probable that a prolonged baking out would eleminate this gas effect.

Duration of Efficiency.

The straight portion of the curve Fig. 4 shows the rate of increase of the vapor pressure passing the trap. The lowest pressure was obtained thirteen days after the Na was introduced. From then on the pressure rose as indicated in a straight line up to the twenty-fifth day, at which time the readings were discontinued. The intercept of this portion of the curve produced back to the ordinate at zero time should give the efficiency of pure Na in the absence of any gas effect. By producing this curve to the right, we see that at the end of one hundred days the pressure will have been augmented by an amount of only 2 divisions, giving a total deflection of 2.8 divisions—that is, a vapor pressure of 8×10^{-7} mm of Hg passing the sodium trap.

FIG. 5. *Table of results.*

Substance in trap	I_e	I_i	I_i/I_e	P_x(mm)	P_o/P_x
Nothing	1,000	132.5	.13250000	[3] $3.\times10^{-3}$	1.0
Cd	1,000	4.0	.00400000	9×10^{-6}	33.3
Na	80,000	.8	.00001000	2.2×10^{-7}	13,250.0
Liquid Air	80,000	[4] .5	.00000625	1.4×10^{-7}	21,200.0

Where

I_e = electron current in scale divisions.

I_i = ion current in scale divisions.

[3] Taken from C. F. Hills curve Fig. 3 for a temperature of 26.3°C; Phys. Rev., *20*, p. 265; 1922.

[4] The ion current of .5 divisions with liquid air on the trap is probably due entirely to the steady evolution of gas from the walls of the ionization gauge.

P_x = pressure in mm of mercury passing trap.
P_o = pressure in mm of mercury without trap.

There are two reasons for concluding that the action of the sodium is a mass action rather than a surface effect. First, in the case of the zinc and gold,[5] the pressure rose quite rapidly, showing that it was a surface effect. There were about six grams of zinc distilled up in a mirror having two or three times the area of the sodium surface. The vapor pressure was lowered approximately 50 per cent and immediately began to rise at such a rate as to reach the normal pressure within some thirty hours. Second, the forming of a new surface of sodium in the trap did not effect the lowering of the vapor pressure. This suggests that the mercury is distributed throughout the mass of the sodium. It seems, then, that we may conclude that the duration of the effectiveness is proportional to the mass of sodium in the trap.

4. CONCLUSIONS

Many laboratories are so situated that it is almost impossible for them to obtain liquid air. The use of sodium as a mercury vapor trap should open to such laboratories an important field for research. In the case of laboratories near big cities, where liquid air may be purchased, the expense and trouble involved in the use of liquid air is quite considerable. One liter of liquid air costs between about $2.00 and $5.00 and lasts not much more than a day. Even when the liquid air is made in the laboratory, the cost is still high when one takes into account a man's time in running the plant. No doubt there are types of experiments in which a liquid air trap would be preferable to the sodium trap, but it is safe to say that for many purposes where we need to prevent mercury vapor passing a given point in a vacuum system, the sodium trap can replace the liquid air trap with advantage. The experiments described in this paper show that satisfactory results have already been obtained with the new method. One may reasonably expect a still better performance as details in technique are worked out.

The writer wishes to thank Professor A. Ll. Hughes for suggesting this research and for his active interest and advice at every step of the experiment.

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ST. LOUIS, MO., JULY 8, 1924.

[5] Wright in The American Journal of Science, *13*, p. 49; 1877, probably had a large surface of gold compared to the rate of diffusion of the mercury vapor. The chances are, too, that the spectrum of mercury would have shown up in a few days of continuous exposure of the gold leaf to the vapor.